## Mathematics and Redistricting - Dr. Kyle Evans and Dr. Adam Giambrone

As you might imagine, mathematics plays a significant role in gerrymandering and partitions of graphs can be used to model or simulate congressional districts.

1) The graphs below show 15 voters and their preferences for Diamond or Star. Despite the fact that the voters' preference and location remain the same, the arrangement of the districts can completely change the outcome. On each graph (map) below, partition (redistrict) the graph into 3 connected shapes (districts) of 5 voters each to match the possible outcomes. If any outcome is not possible, explain why.
a) Using the information above, what is the minimum number of voters that either or $\star$ needs to win:
(i) every district?
(ii) the majority of districts?

b) Desired outcome:

- wins all 3 districts
$\qquad$ $\star$, $\qquad$

c) Desired outcome:
- wins 2 districts, $\star$ wins 1

District 1: $\qquad$ - $\qquad$

District 2: $\qquad$ - $\qquad$

District 3: $\qquad$ $\star$ $\qquad$

| $\star$ | $\diamond$ | $\diamond$ |
| :---: | :---: | :---: |
| $\star$ | $\star$ | $\star$ |
| $\star$ | $\star$ | $\star$ |
| $\star$ | $\star$ | $\star$ |
| $\star$ | $\star$ | $\star$ |

d) Desired outcome:
$\star$ wins 2 districts, wins 1
District 1:__
District 2:__
District $3:$

e) Desired outcome:
$\star$ wins all 3 districts

District 1: $\qquad$ *,__ $\star$

District 2: $\qquad$ $\star$,__ $\star$

District 3: $\qquad$ - $\qquad$
f) Which of the outcomes in (b), (c), (d), and (e) do you think is the "most fair" and why?
2) The graphs now show 50 voters and their preferences for Diamond or Star. On each graph (map) below, partition (redistrict) the graph into 5 connected shapes (districts) of 10 voters each to match the possible outcomes. Label each district in the graph. If any outcome is not possible, explain why.
a) Using the information above, what is the minimum number of voters that either or needs to win:
(i) every district?
(ii) the majority of districts?

| $\star$ | $\star$ | $\star$ | $\checkmark$ | $\checkmark$ | $\star$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\star$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\star$ | $\star$ | $\star$ | $\checkmark$ |
| $\checkmark$ | $\checkmark$ | $\star$ | $\star$ | $\star$ | $\star$ | $\checkmark$ | $\star$ | $\star$ | $\star$ |
| $\star$ | $\checkmark$ | $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\star$ | $\checkmark$ | $\checkmark$ | $\star$ |
| $\star$ | $\star$ | $\bullet$ | $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\star$ | $\checkmark$ | $\star$ |

b) Desired outcome: $\star$ wins the majority of districts

District 1: $\qquad$ - $\qquad$ $\star$

District 2: $\qquad$ $\checkmark$ $\qquad$ $\star$

District 3: $\qquad$ $\bullet$, $\qquad$ _

District 4: $\qquad$ $\diamond$, $\qquad$ $\star$

District 5: $\qquad$ - $\qquad$ $\star$

| $\star$ | $\star$ | $\star$ | $\checkmark$ | $\checkmark$ | $\star$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\star$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\star$ | $\star$ | $\star$ | $\checkmark$ |
| $\checkmark$ | $\checkmark$ | $\star$ | $\star$ | $\star$ | $\star$ | $\checkmark$ | $\star$ | $\star$ | $\star$ |
| $\star$ | - | $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\star$ | - | $\checkmark$ | $\star$ |
| $\star$ | $\star$ | $\checkmark$ | $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\star$ | $\checkmark$ | $\star$ |

c) Desired outcome: wins the majority of districts

District 1: $\qquad$ $\bullet$, $\qquad$ $\star$

District 2: $\qquad$ $\checkmark$, $\qquad$ $\star$

District 3: $\qquad$ $\diamond$, $\qquad$ $\star$

District 4: $\qquad$ $\star$,___

District 5: $\qquad$ $\checkmark$, $\qquad$ $\star$
d) Which of the outcomes in (b) and (c) do you think is "more fair" and why?
3) The graphs now show 56 voters and their preferences for Diamond, Star, or Circle. On each graph (map) below, partition (redistrict) the graph into 5 connected shapes (districts) of 11 or 12 voters each to match the possible outcomes. Label each district in the graph.
a) Using the information above, what is the minimum number of voters that either $\star$ or needs to win: (i) every district?
(ii) the majority of districts?

| $\star$ | $\star$ | $\star$ | $\checkmark$ | $\checkmark$ | $\star$ | $\checkmark$ | $\checkmark$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\bullet$ | $\star$ | $\star$ | $\star$ |
| $\checkmark$ | $\checkmark$ | $\star$ | $\bullet$ | $\star$ | $\star$ | $\checkmark$ | $\star$ |
| $\star$ | $\checkmark$ | $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\star$ | $\checkmark$ |
| $\star$ | $\star$ | $\checkmark$ | $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\checkmark$ |
| $\checkmark$ | $\star$ | $\star$ | $\checkmark$ | $\checkmark$ | $\star$ | $\star$ | $\checkmark$ |
| $\star$ | $\checkmark$ | $\star$ | $\star$ | $\star$ | $\checkmark$ | $\checkmark$ | $\star$ |

b) Desired outcome: $\star$ wins as many districts as possible

District 1: $\qquad$ - $\qquad$ $\star$, $\qquad$ -

District 2: $\qquad$ $\star$,__ $\star$, $\qquad$ -

District 3: $\qquad$ $\star, \ldots \quad \star$, $\qquad$ -

District 4: $\qquad$ $\star$,__ $\star$, $\qquad$ -

District 5: $\qquad$ - $\qquad$ $\star$, $\qquad$ -

| $\star$ | $\star$ | $\star$ | $\bullet$ | $\checkmark$ | $\star$ | $\bullet$ | $\checkmark$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\star$ | $\star$ |
| $\checkmark$ | $\checkmark$ | $\star$ | $\bullet$ | $\star$ | $\star$ | $\checkmark$ | $\star$ |
| $\star$ | $\checkmark$ | $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\star$ | $\checkmark$ |
| $\star$ | $\star$ | $\checkmark$ | $\checkmark$ | $\star$ | $\checkmark$ | $\star$ | $\checkmark$ |
| $\checkmark$ | $\star$ | $\star$ | $\checkmark$ | $\checkmark$ | $\star$ | $\star$ | $\checkmark$ |
| $\star$ | $\checkmark$ | $\star$ | $\star$ | $\star$ | $\checkmark$ | $\checkmark$ | $\star$ |

c) Desired outcome: wins as many districts as possible

District 1: $\qquad$ $\star$,__ $\star$, $\qquad$ -

District 2: $\qquad$ $\star$,__ $\star$, $\qquad$ -

District 3: $\qquad$ $\star$, $\star$, $\qquad$

District 4: $\qquad$ $\star, \ldots \quad \star$, $\qquad$

District 5: $\qquad$ $\checkmark$, $\qquad$ $\star$, $\qquad$
d) Which of the outcomes in (b) and (c) do you think is "more fair" and why?
e) In the graph (map) with 56 voters, what distribution of districts do you think is the "most fair"? (You are not limited to your answers in parts (a) and (b), other outcomes are possible.) Partition (redistrict) the graph into 5 connected shapes (districts) of 11 or 12 voters each so that it has the outcome that you decided is the "most fair." Label each district in the graph.

|  |  | * |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| * |  |  |  |  |  |  |
|  |  |  |  |  | - |  |
|  |  | * |  | $\star$ | * |  |
|  |  | * |  | * |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

Desired outcome: "Most fair"
District 1:___ $\quad \star \quad \_\quad \_$, Winner:
District 2: ___ $\quad \ldots \quad \star, \ldots$, Winner:
District 3: ___ $\quad \ldots \quad \star, \ldots \quad$, Winner:
District 4: $\qquad$ $\star$, _ $\star$, $\qquad$ -, Winner: $\qquad$

District 5: $\qquad$ $\bullet$, $\qquad$ $\star$, $\qquad$ -, Winner: $\qquad$

How did you decide that this outcome was the "most fair"?

