Redistricting / Gerrymandering

Vocabulary

Redistricting – the redrawing of maps / districts so that each district contains approximately the same number of people, required every 10 years and last occurred in 2012 following the 2010 U.S. Census, will next occur in 2022 and all federal and state districts will be redrawn based on 2020 U.S. Census populations

Gerrymandering – the practice of political parties in power redrawing the district lines to maximize the representation of their party

Racial Gerrymandering – the (illegal) practice of drawing district lines to reduce the impact of minority voters by limiting their ability to elect candidates of their choice (unconstitutional: Voting Rights Act of 1965)

Partisan Gerrymandering – the practice of drawing district lines to maximize a political party's advantage, this is legal and in March 2019, the Supreme Court will hear cases to determine its constitutionality

Redistricting Principles (*required)

*(1) to ensure the "one person, one vote" principle, all congressional and state legislature districts must contain roughly the same number of people

*(2) all districts must be **contiguous**, meaning that they are one connected shape

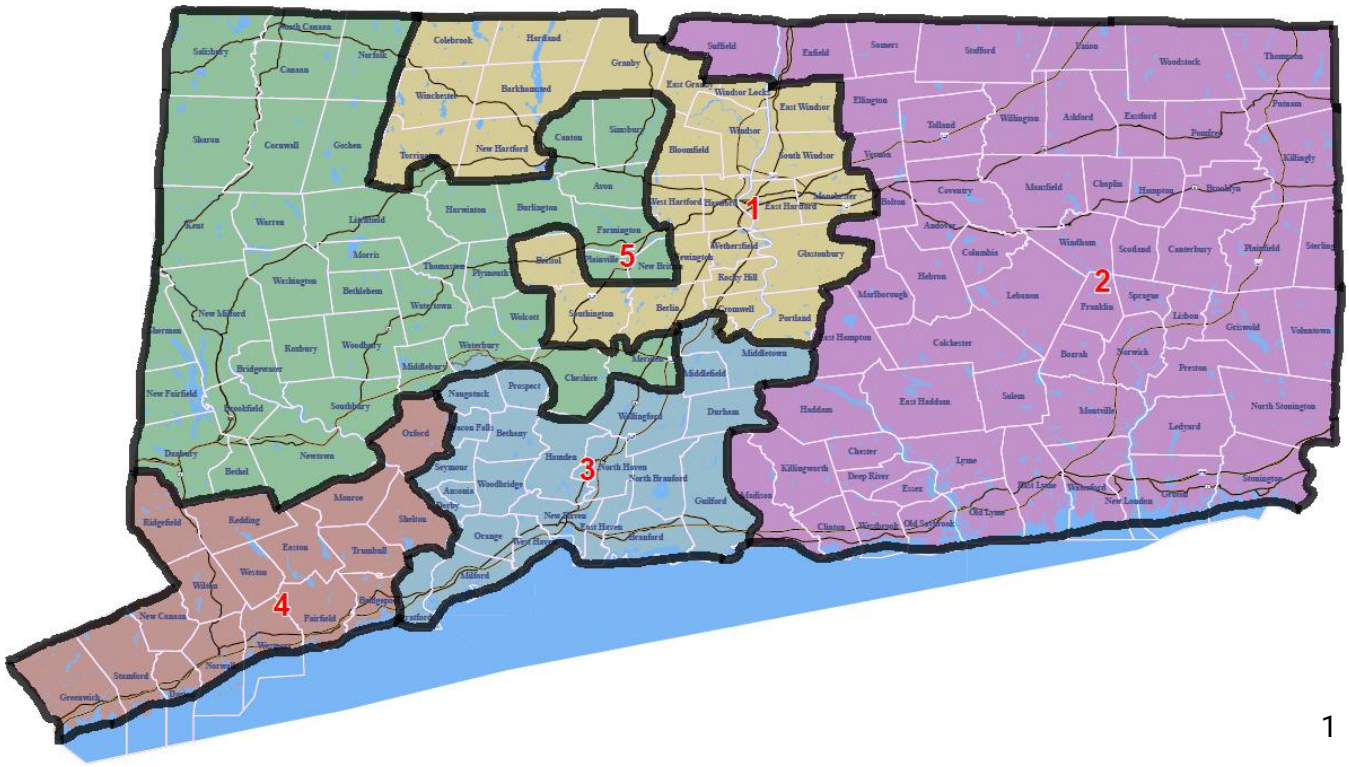
*(3) districts must abide by racial fairness in accordance with the Voting Rights Act of 1965

(4) districts should be **compact**, meaning that they have "nice and tidy" shapes

(5) districts should be fair to each party, with representation being roughly proportional to each party's voters

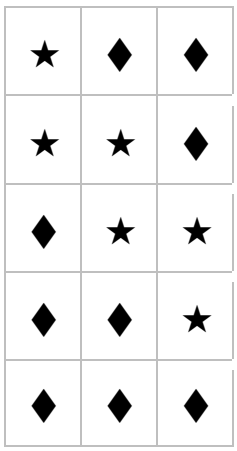
(6) districts should preserve existing political communities rather than separate them into different districts

Below is the current map of Connecticut's 5 Congressional Districts. For its state legislature, Connecticut has 151 districts for their House of Representatives and 36 districts for their Senate.



As you might imagine, mathematics plays a significant role in redistricting / gerrymandering and partitions of grids can be used to model or simulate districts of voters.

The grids below show 15 voters and their preferences for Diamond or Star. Despite the fact that the voters' preference and location remain the same, the arrangement of the districts can completely change the outcome. On each grid (map) below, partition (redistrict) the grid into 3 connected shapes (districts) of 5 voters each to match the possible outcomes.

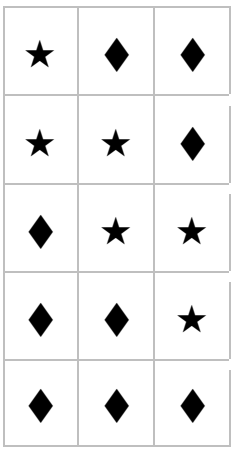


Goal: ♦ wins all 3 districts

District 1: ____ ♦, ____ ★

District 2: ____ ♦, ____ ★

District 3: ____ ♦, ____ ★

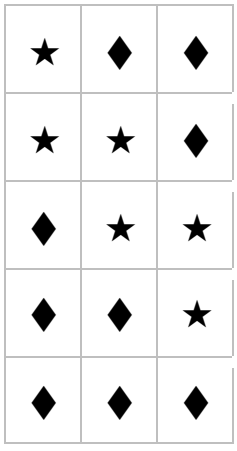


Goal: ♦ wins 2 districts
★ wins 1 district

District 1: ____ ♦, ____ ★

District 2: ____ ♦, ____ ★

District 3: ____ ♦, ____ ★

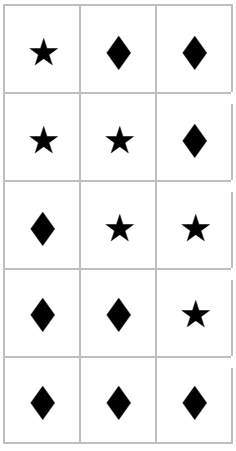


Goal: ★ wins 2 districts
♦ wins 1 district

District 1: ____ ♦, ____ ★

District 2: ____ ♦, ____ ★

District 3: ____ ♦, ____ ★



Goal: ★ wins all 3 districts

District 1: ____ ♦, ____ ★

District 2: ____ ♦, ____ ★

District 3: ____ ♦, ____ ★

What is the minimum number of voters that either ♦ or ★ needs to win:
(a) every district? (b) the majority of districts?

Which of the outcomes above do you think is the "most fair"?

Gerrymandering Strategies

Packing – the practice of a party in power placing as many voters of the opposing party into as few districts as possible to maximize the number of districts won by the party in power

Cracking – the practice of a party in power splitting up voters of the opposing party into many districts to obtain a majority in as many districts as possible

Redistricting Goals

While gerrymandering focuses on political parties maximizing their advantage, neutral individuals (independent commissions) may have other goals when redistricting:

Proportional Representation: While nothing is guaranteed because actual representation depends on election results, mapmakers can use existing data to draw districts that are likely to give each party their share of districts based on their share of voters in the state.

Competitive Elections: Another goal is to increase elections that would be competitive on Election Day by using existing data to draw districts that would expect to have close elections and increase the importance of votes.

Page 2 examples

For each of the different outcomes on the grids from page 2, identify the relevant gerrymandering strategy (packing / cracking / both? / neither?) and whether the map (grid) seems to have been redistricted for a partisan advantage, proportional representation, or competitive elections. (more than one of these? none?)

Map	Gerrymandering Strategy	Redistricting Goal
♦ wins all 3 districts		
♦ wins 2 districts, ★ wins 1		
★ wins 2 districts, ♦ wins 1		

The grids now show 50 voters and their preferences for Diamond or Star. On each grid (map) below, partition (redistrict) the grid into 5 connected shapes (districts) of 10 voters each to match the possible outcomes.

First, what is the minimum number of voters that either ♦ or ★ needs to win:
(a) every district? (b) the majority of districts?

★	★	★	♦	♦	★	♦	♦	♦	★
♦	★	♦	★	♦	★	★	★	★	♦
♦	♦	★	★	★	★	♦	★	★	★
★	♦	♦	★	♦	★	★	♦	♦	★
★	★	♦	♦	★	♦	★	★	♦	★

Goal: ★ wins as many districts as possible

District 1: ____ ♦, ____ ★

District 2: ____ ♦, ____ ★

District 3: ____ ♦, ____ ★

District 4: ____ ♦, ____ ★

District 5: ____ ♦, ____ ★

★	★	★	♦	♦	★	♦	♦	♦	★
♦	★	♦	★	♦	★	★	★	★	♦
♦	♦	★	★	★	★	♦	★	★	★
★	♦	♦	★	♦	★	★	♦	♦	★
★	★	♦	♦	★	♦	★	★	♦	★

Goal: ♦ wins as many districts as possible

District 1: ____ ♦, ____ ★

District 2: ____ ♦, ____ ★

District 3: ____ ♦, ____ ★

District 4: ____ ♦, ____ ★

District 5: ____ ♦, ____ ★

For each map above, note the gerrymandering strategy (or strategies) used. Then decide which of these two maps you think is fairer, and what the "most fair" outcome would be for this map's voters and districts.

What measures exist to quantify the "fairness" of a map and its districts to test for possible gerrymandering?

The first measure is the **efficiency gap**, which analyzes the extent to which parties "wasted votes" with the idea being that the more votes a party "wastes", the more biased the map is towards the other party.
Note: This concept was first published in 2015 by Nicholas Stephanopoulos, Professor at the University of Chicago Law School, and Eric McGhee, Research Fellow at the Public Policy Institute of California.

Wasted Vote – A vote is considered "wasted" if it meets either of these criteria:
(1) for a winning party, any votes cast beyond the number needed to win (additional votes were not needed)
(2) for a losing party, any votes cast (these votes were not needed since they did not lead to a win)

Efficiency Gap – the difference in wasted votes divided by the total number of votes

$$\text{Efficiency Gap} = \frac{(\bigstar \text{ Wasted Votes} - \blacklozenge \text{ Wasted Votes})}{\text{Total Votes}} \times 100$$

Example: Suppose a town has 500 voters divided into 5 districts of 100 voters each with the results of a vote for local representation shown in the table below.

District	★ votes	♦ votes
1	75	25
2	60	40
3	43	57
4	48	52
5	49	51
Total		

★ wasted	♦ wasted

Summary	★	♦
Districts Won (%)		
% of Voters		
Wasted Votes		
Net Wasted Votes	★ / ♦	
Efficiency Gap	★ / ♦	
Benefit?		

The authors wrote that maps of congressional districts (by state) should be ruled unconstitutional if one party wins at least two more seats than expected. In addition, they wrote that maps of state legislature districts should be ruled unconstitutional if the efficiency gap is 8% or more.

By this metric alone, would the example above be deemed "unconstitutional" (unfairly biased for one party)? What factors does the efficiency gap ignore?

Go back to your maps drawn on pages 2 and 4 and calculate the efficiency gap for all 5 of your maps.

Page 2

♦ wins all 3 districts				
District	★	♦	★w.v	♦w.v
1				
2				
3				
Total			Net:	
Efficiency Gap →			★ / ♦	

♦ wins 2, ★ wins 1 district				
District	★	♦	★w.v	♦w.v
1				
2				
3				
Total			Net:	
Efficiency Gap →			★ / ♦	

★ wins 2, ♦ wins 1 district				
District	★	♦	★w.v	♦w.v
1				
2				
3				
Total			Net:	
Efficiency Gap →			★ / ♦	

What can be concluded from these results?

Page 4

★ wins as many as possible				
District	★	♦	★w.v	♦w.v
1				
2				
3				
4				
5				
Total			Net:	
Efficiency Gap →			★ / ♦	

♦ wins as many as possible				
District	★	♦	★w.v	♦w.v
1				
2				
3				
4				
5				
Total			Net:	
Efficiency Gap →			★ / ♦	

What can be concluded from these results?

While the efficiency gap has been the most used mathematical metric in court cases involving possible gerrymandering, the approach is not without flaws. Complete the two tables below to determine the efficiency gaps of a non-competitive districts example and a competitive districts example.

Non-competitive				
District	★	♦	★w.v	♦w.v
1	8	1		
2	8	1		
3	8	1		
4	8	1		
5	8	1		
6	8	1		
7	8	1		
Total			Net:	
Efficiency Gap →				★ / ♦

Competitive				
District	★	♦	★w.v	♦w.v
1	10	9		
2	10	9		
3	10	9		
4	9	10		
5	9	10		
Total			Net:	
Efficiency Gap →				★ / ♦

In the non-competitive table, what conclusion can be drawn?

Why does this claim not make sense in this example?

In the competitive table, what conclusion can be drawn?

Why does this claim not make sense in this example?

One issue with the efficiency gap is that simple math shows that there are a certain number of votes which must be wasted by at least one party. These votes shouldn't count against you so Jeffrey Barton, a math professor at UT-Austin proposed the *modified efficiency gap* in 2018.

Use the examples from page 7 to complete the following information which will demonstrate the concept of the modified efficiency gap.

Non-competitive

★ is the majority party and won all 7 districts, but only needed _____ voters to win all 7 districts. By using all of their voters to win all 7 districts, ★ used _____ more voters than they needed to win all 7 districts.

♦ is the minority party and didn't win any districts, but they could have won _____ districts at most. If ♦ did win that many districts, they would need to use _____ voters and would have _____ voters left over.

★ Wasted Votes (from page 7) =

★ Must Waste (from above) =

★ Actually Wasted votes =

♦ Wasted Votes (from page 7) =

♦ Must Waste (from above) =

♦ Actually Wasted votes =

The modified efficiency gap is the same formula as the efficiency gap but compares votes that were actually wasted instead of all wasted votes.

Modified Efficiency Gap =

Conclusion:

Competitive

★ is the majority party and won 3 districts, but could have won _____ districts at most. If ★ did win that many districts, they would need to use _____ voters and would have _____ voters left over.

♦ is the minority party and won 2 districts, but could have won _____ districts at most. If ♦ did win that many districts, they would need to use _____ voters and would have _____ voters left over.

★ Wasted Votes (from page 7) =

★ Must Waste (from above) =

★ Actually Wasted votes =

♦ Wasted Votes (from page 7) =

♦ Must Waste (from above) =

♦ Actually Wasted votes =

Modified Efficiency Gap =

Conclusion:

The efficiency gap measures the distribution of voters in each district, but ignores many other factors, such as the physical shape of the district. There are many different methods used to measure "how nice" a district's shape is, otherwise known as **compactness**.

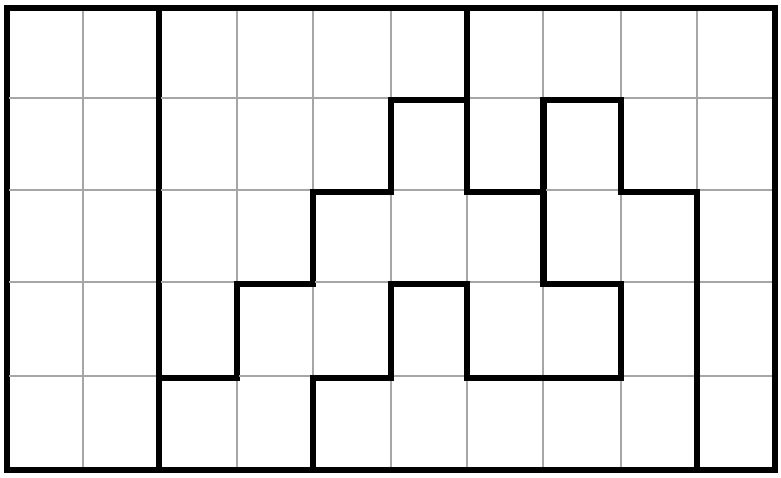
One way to measure the compactness of a district is to look at a ratio of its perimeter and area. A larger perimeter needed to cover a smaller area would suggest potential gerrymandering (such as MD-3 on right).

We also want a ratio that can be compared across any districts, which is the idea behind the **Polsby-Popper Test**: (concept originally developed by a paleontologist to measure the roundness of sand grains in 1927, refined and first used in redistricting in 2000 by Arizona)

$$\text{Polsby-Popper score} = \frac{\text{Area of district}}{\text{Area of square with the same perimeter as the district}}$$

This answer must be a number between 0 and 1 and tells us how far off the district shape is from a square. *Note:* This method actually uses circles, but we will use squares because our examples are grids.

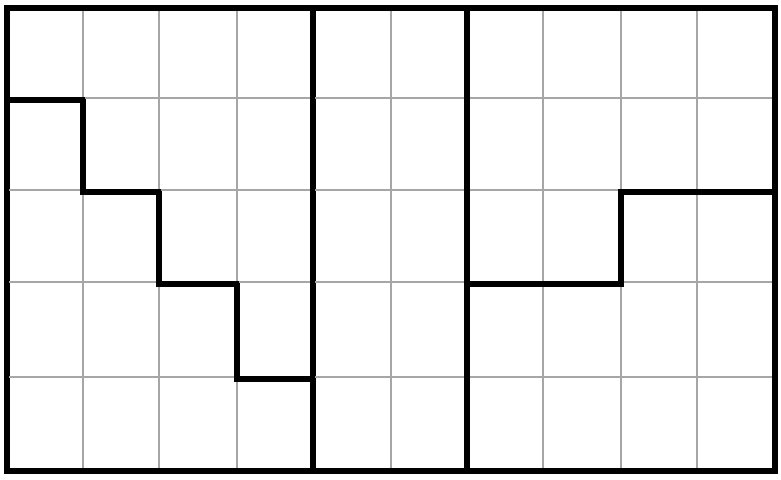
For example, consider the districting plan below:



District	Area	Perimeter	Area of Square w/ same perimeter	Polsby-Popper
1				
2				
3				
4				
5				

Is there reason to believe that this map has been gerrymandered?

Now consider the same map with a different districting plan:



District	Area	Perimeter	Area of Square w/ same perimeter	Polsby-Popper
1				
2				
3				
4				
5				

Is there reason to believe that this map is gerrymandered?

How would you claim that one of the two districting plans is "more fair" based on this compactness test?

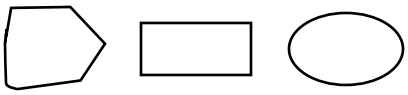
What factors does compactness ignore?

What shape district would score perfectly (Polsby-Popper score = 1) using this test?
Draw the shape of a district that would have a low score using this test.

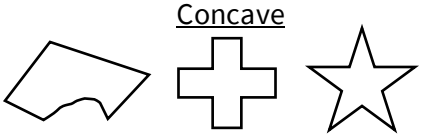
Another test used to measure compactness uses the **convex hull** of a district's shape, which is the smallest *convex* shape that surrounds the district.

Brief geometry detour: a shape is **convex** if every line that connects two points inside the shape is entirely inside the shape and if any part of any of these lines leaves the shape, then the shape is **concave**
(Another way to think about this is to look for any "caved in" angles.)

Convex



Concave

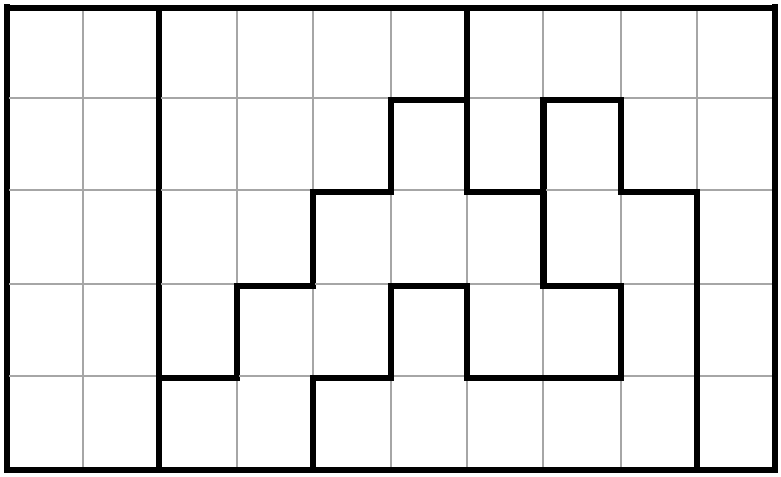


In reality, the convex hull of a shape is the shape you would get when placing a rubber band around the shape, but since we are using grids, our definition will be:

Convex Hull – the <u>smallest rectangle</u> which contains the district, and to use this to measure compactness:

$$\text{Convex Hull Test/score (using area)} = \frac{\text{Area of District}}{\text{Area of District's Convex Hull}}$$

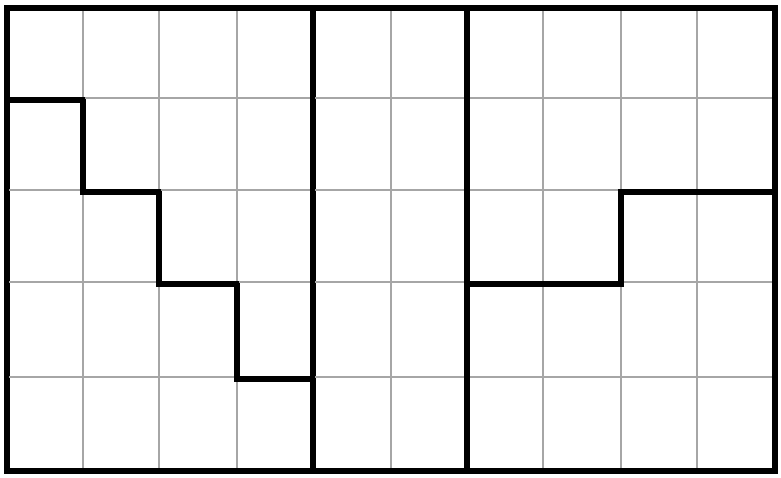
Returning to the same examples from pages 9 and 10, let's now measure compactness using the convex hull:



District	Area	Area of Convex Hull	Convex Hull Test
1			
2			
3			
4			
5			

Are there any significant difference between each district's Polsby-Popper score and their Convex Hull score?

Using the example from page 8 now with convex hull:



District	Area	Area of Convex Hull	Convex Hull Test (with areas)
1			
2			
3			
4			
5			

For this plan, are there any significant differences between each district's Polsby-Popper score and their Convex Hull score?

What shapes of districts would score perfectly (Convex Hull score = 1) using this test?
Draw the shape of a district that would have a low score using this test.

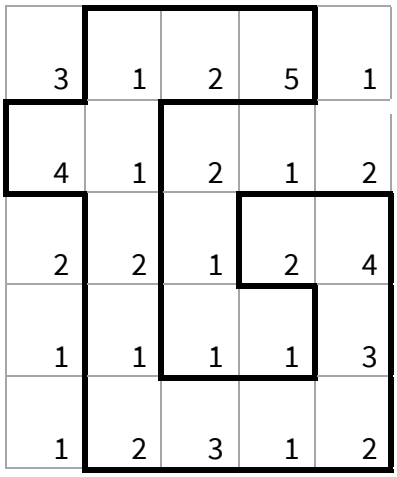
Can you think of shapes that would score really well on one of these two tests (Polsby-Popper / Convex Hull) and not as well on the other test?

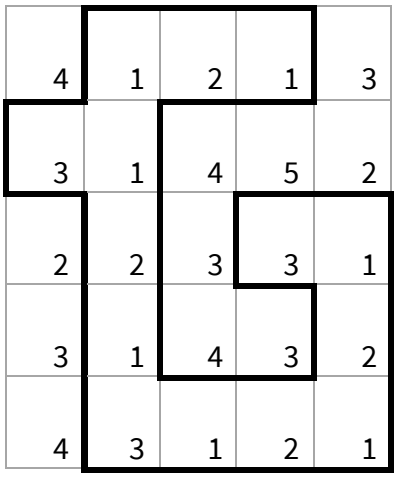
The concept of the convex hull can be applied to compare areas or to compare *populations*.

The convex hull is still the smallest rectangle that contains the district, and using populations in the ratio as opposed to areas provides another test for compactness:

$$\text{Convex Hull Test/score (using population)} = \frac{\text{Population of District}}{\text{Population of District's Convex Hull}}$$

For example, the two district shapes below are identical but the population distributions are different. (The overall map would be much larger, and this is just one district in a larger picture.)





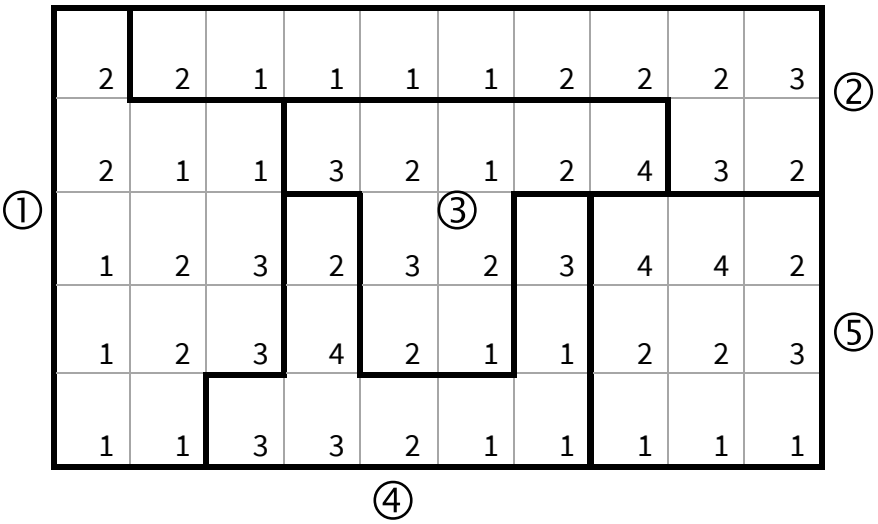
Map	Population	Population of Convex Hull	Convex Hull Test (w/ populations)	Area	Area of Convex Hull	Convex Hull Test (with areas)
Left						
Right						

Using the Convex Hull Test (using populations), which map is more likely to have been gerrymandered?

What shapes of districts / distribution of populations would score perfectly (Convex Hull population score = 1) using this test? What shapes / distribution of populations would score low?

Can you come up with an instance of a district having a high Convex Hull area score and a low Convex Hull population score and vice versa?

For the following map of districts below, apply all 3 compactness tests and compare the scores within a district and across districts.
Note: There are 100 total voters and each district contains 20 voters.



D#	Area	Perim.	Pop.	Area of Square w/ same Per.	Polsby-Popper Test	Area of Convex Hull	Convex Hull Test (area)	Population of Convex Hull	Convex Hull Test (population)
1									
2									
3									
4									
5									

Interpretation: Do you have any reason to believe that this map has been gerrymandered? Which district or districts (if any) are most questionable by these scores?

What is needed to further interpret the "fairness" of this districting plan?

Compactness Criteria: We have used three different tests to measure the compactness of a district, but how can these numbers be interpreted? Different researchers have come up with similar "criteria" for the use of any compactness measures.

(1) There is no single threshold value which determines when a district is no longer compact. The measures should be used as a comparison rather than the numbers by themselves.
(2) Compactness measures should be applied to an entire districting plan, not just a single district.
(3) Comparisons should not be made across states, simply between districts and plans within states.
(4) Any compactness test should measure the shape of the district, not the size of the district.
(5) Multiple compactness tests should be used whenever possible.
(6) No districting plan should be judged solely by compactness tests, other criteria should also be used.

Redistricting Understanding

Overall Concepts

- [ ] Redistricting vs. Gerrymandering
- [ ] Partisan Gerrymandering vs. Racial Gerrymandering
- [ ] Redistricting Principles
- [ ] Gerrymandering strategies (packing / cracking)
- [ ] Redistricting goals (proportional representation and competitive elections)
- [ ] The number of voters needed to win a district or group of districts
- [ ] Drawing a districting map to achieve various outcomes
- [ ] Prison Gerrymandering

Efficiency Gap

- [ ] The concept of wasted votes
- [ ] The calculation of the efficiency gap
- [ ] The interpretation of the efficiency gap

Compactness

- [ ] Polsby-Popper test
- [ ] Convex Hull test (using area)
- [ ] Convex Hull test (using population)
- [ ] Compactness criteria / interpretation of compactness scores