Mathematics and Redistricting

Trinity College, Fall 2019 (Kyle Evans)
(Re)Apportionment

United States Constitution requires the House of Representatives to be (re)apportioned every 10 years
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...but how?

Constitution:

• “Each state shall have at least one Representative”

• “The number of Representatives shall not exceed one for every 30,000”
(Re)Apportionment

Suppose you had to assign 10 representatives for the following four states based on their populations:

- State A: 25,000
- State B: 35,000
- State C: 18,000
- State D: 22,000

Total: 100,000 people
(Re)Apportionment

Suppose you had to assign 10 representatives for the following four states based on their populations:

- State A: 25,000 → 2 reps? 3 reps?
- State B: 35,000 → 3 reps? 4 reps?
- State C: 18,000 → 2 reps?
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Math question: How do we translate fractional amounts to whole number representatives?
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Add up the lower quotas, how many seats remain?
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• Step 4: Fractional Part = number following the decimal of the standard quota
Assign states extra seats starting with largest fractional part

Final representatives = (Lower Quota) or (Lower Quota + 1)
Jefferson’s Method

1792 – George Washington vetoed Hamilton’s Method in favor in a method supported (sponsored) by Thomas Jefferson

Fun fact: This was the first presidential veto in U.S. history!

At this point, the total number of representatives was not fixed and changed every 10 years
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• Step 2: Choose a Modified Divisor (slightly less than the standard divisor) and for each state: Modified Quota = (State Population) ÷ (Modified Divisor)
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- Step 1: Calculate the Standard Divisor, Standard Quotas, and Lower Quotas.
- Step 2: Choose a Modified Divisor (slightly less than the standard divisor) and for each state: Modified Quota = (State Population) ÷ (Modified Divisor)
- Step 3: Check if the new Lower Quotas add up to the desired number of representatives. If not, try a different Modified Divisor.
  - If you are still have representatives left over, choose a slightly lower modified divisor.
  - If you assigned too many representatives, choose a slightly higher modified divisor.
Jefferson’s Method

• Used from 1792–1832

• 1832 – New York’s standard quota = 38.59
  New York’s number of rep’s = 40
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Example of an upper quota violation showing that method favors states with larger populations
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Check how many representatives have been apportioned.

- Step 2: Choose a Modified Divisor (adjust for needing to gain or lose reps) and for each state: Modified Quota = (State Population) ÷ (Modified Divisor)
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  • If you need to gain representatives, choose a slightly lower modified divisor.
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Mathematical Compromise and Confusion

- 1852-1892: Adjusted total number of Representatives so that Hamilton’s Method and Webster’s Method agreed and produced the same apportionment.

- 1872 – many new states admitted to Union and the new apportionment didn’t agree with either method and contributed to controversial Hayes vs. Tilden 1876 Presidential election.
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• 1882 – discovery of the Alabama paradox – using Hamilton’s Method someone realized with 299 Rep’s AL would get 8, but with 300 Rep’s AL would get 7.

• 1901 – Alabama paradox led to a proposal minimizing Rep’s for ME and CO and Hamilton’s Method was no longer used.

• Webster’s Method used from 1901–1941.
Huntington-Hill Method

• 1941 – Congress passes a law setting 435 as fixed number of representatives and using a new method created by a mathematician and statistician

• Step 1: Calculate the Standard Divisor, Standard Quotas, and both the Lower and Upper Quotas (always round up)
Huntington-Hill Method

• 1941 – Congress passes a law setting 435 as fixed number of representatives and using a new method created by a mathematician and statistician.

• Step 1: Calculate the Standard Divisor, Standard Quotas, and both the Lower and Upper Quotas (always round up).

• Step 2: Round Standard Quotas according to the Geometric Mean of the lower and upper quotas.

\[
\text{Geometric Mean of } a \text{ and } b = \sqrt{ab}
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Check how many representatives have been apportioned.
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Check how many representatives have been apportioned.

- Step 3: Choose a Modified Divisor and round Modified Quotas according to the geometric means until the correct number of representatives are assigned.
Huntington–Hill Method

• This is the method still in use today

• (1991) *Montana v. U.S. Dep’t of Commerce* – Supreme Court ruled that the Huntington–Hill method is constitutional

• Just requires state populations and a simple Excel table!
Current Map by number of Representatives