Mathematics and Redistricting

Trinity College, Spring 2021 (Kyle Evans)
H.R. 1 – For The People Act of 2021

• Passed the U.S. House on March 3, 2021 (220-210)

• Voting
  • Automatic voter registration
  • 15 days of early voting
  • Limitations on voter purging
  • Expand mail-in voting

• Redistricting (starting with 2030 Census)
  • Independent commissions of citizens in all states (5 Dem., 5 Rep., 5 Ind.)
  • Transparent to public, open comment period
  • Not allowed to use party affiliation data
H.R. 1 – For The People Act of 2021

- Passed the U.S. House on March 3, 2021 (220-210)
- Campaign finance reform
  - Public information on donations to super PACs
  - Public information on Facebook/Twitter political ads
- Ethics rules
  - Members of Congress can’t use taxpayer money to settle discrimination cases
  - Additional oversight on lobbyists
- Tax returns
  - President, Vice President, and candidates must release 10 years of tax returns
H.R. 1 – For The People Act of 2021

- Passed the U.S. House on March 3, 2021 (220-210)
- Unlikely to pass Senate with current filibuster rules

- Movement among states in the opposite direction
- Georgia House passed last week (Senate passed their version yesterday)
  - No automatic voter registration
  - No early voting on Sundays
  - Additional ID requirements
  - Limit on ballot drop boxes
  - Can’t give food or water to voters standing in line
What makes a district map fair?

The mathematical community has offered various quantitative measures and approaches to attempt to classify the “fairness” of any district map:

• Efficiency gap (based on party of voters)
• Compactness (based on geometry of districts)
• Algorithms to generate new maps
• Simulations of elections
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What is the efficiency gap?

• A quantitative measure to define the extent of partisan gerrymandering

• Impacted the initial court ruling in *Gill v. Whitford* (2016)
  • Democrats had “trifecta” in before 2010 elections, Republicans had trifecta after 2010 elections and got to control redistricting process

• District Court ruled that Wisconsin had to redraw State House map by 2017

• Appeal to Supreme Court → sent back to lower courts to decide after SC ruled that partisan gerrymandering was to be determined by the states
What is the efficiency gap?

Developed by Stephanopoulos / McGhee in 2014

Based on the concept of “wasted votes” and designed to measure the extent of packing and cracking of voters

• For a winning party, any votes above those needed to win are “wasted”
• For a losing party, any votes are “wasted”
• Calculation is based on which party “wastes” more votes than the other
• Efficiency gap = (difference in wasted votes) / total votes (x 100%)
**What is the efficiency gap?**

Example:

<table>
<thead>
<tr>
<th>District</th>
<th>⭐ wasted</th>
<th>♦ wasted</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
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![Diagram](image)
What is the efficiency gap?

Example:

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![Diagram showing the efficiency gap example](image)
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<td>3</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
<td>9</td>
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 EG = (11−9)/45 = 4% in favor of ♦ (less votes wasted)
Interpreting the efficiency gap

What does the final calculation mean?
• 0% means each wasted the same number of votes → perfectly fair
• The higher the %, the more unfair
• When multiplied by # of districts, % gives us how much of a seat advantage
• From example: 4% × 5 districts = 0.2 seat advantage for ♦

What is considered “too unfair”?
• Congressional district plans: +2 (or more) seats
• State district plans: +8% (or more)
Efficiency gap example

$$EG = \frac{(14 - 6)}{45} = 18\%$$ in favor of ★ → +1 seat advantage
When all districts have the same number of voters, the efficiency gap has a shortcut formula:

$$\text{Efficiency Gap} = (\% \text{ of Districts won} - 50\%) - 2 \times (\% \text{ of Votes} - 50\%)$$
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In example #1, ★ wins $3/5 = 60\%$ of districts and has $26/45 = 58\%$ of votes

$$\text{EG} = (60\% - 50\%) - 2 \times (58\% - 50\%) = -6\% \rightarrow 6\% \text{ advantage for } \blacklozenge$$
Alternative Formula

When all districts have the same number of voters, the efficiency gap has a shortcut formula:

Efficiency Gap = (% of Districts won – 50%) – 2 x (% of Votes – 50%)

In example #1, ★ wins 3/5 = 60% of districts and has 26/45 = 58% of votes
EG = (60% – 50%) – 2 x (58% – 50%) = -6% → 6% advantage for ♦

In example #2, ★ wins 4/5 = 80% of districts and has 26/45 = 58% of votes
EG = (80% – 50%) – 2 x (58% – 50%) = 14% advantage for ★
Efficiency Gap = (% of Districts won – 50%) – 2 x (% of Votes – 50%)

If one party receives 60% of the votes, then according to the efficiency gap formula, the fairest map is one in which that party wins what % of districts?
**Alternative Formula**

Efficiency Gap = ($\%$ of Districts won − 50%) − 2 x ($\%$ of Votes − 50%)

If one party receives 60% of the votes, then according to the efficiency gap formula, the fairest map is one in which that party wins what % of districts?

\[
(x - 50\%) - 2 \times (60\% - 50\%) = 0
\]

\[
x = 70\% \text{ of districts}
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Alternative Formula

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If one party receives 60% of the votes, then according to the efficiency gap formula, the fairest map is one in which that party wins what % of districts?

\[(x - 50\%) - 2 \times (60\% - 50\%) = 0\]

\[x = 70\% \text{ of districts}\]

In other words, proportional representation isn't always considered fair!
Efficiency gap examples

What is similar and different about the two district plans below?
We conclude that the efficiency gap of an entire districting plan only depends on the number of districts won, not how they are won.

The two examples have 3 district wins in very different ways, but the overall number of wasted votes are identical.

This begs the question about the usefulness of the formula...
Criticisms of the efficiency gap

• Uses election results, but redistricting comes before elections
• “Judging fairness from 1 election is like judging fairness of a coin after 1 flip”
• Wasted votes would be like saying “wasted runs” in baseball
• +2 seat advantage much more likely in larger states
• Ignores competitive elections
• Why is 75/25 the “most fair” district outcome?
• Chief Justice Roberts: “sociological gobbledygook"
• Doesn’t always make sense mathematically!
Efficiency gap in practice

- 2018 – Missouri became the first state to amend its Constitution to explicitly address the efficiency gap, stating that legislative plans must aim to get this value as close to zero as possible
  - Ballot initiative that passed also included a new position of “state demographer”
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  • Ballot initiative that passed also included a new position of “state demographer”

• 2020 – Missouri voters passed a new ballot initiative that essentially eliminated redistricting reform from 2018
  • State demographer replaced with political commissions
  • Attempt to only count voting-age adults
  • Public transparency and input still exist
Mathematical fixes

“Relative” efficiency gap

• Instead of number of wasted votes, compare proportions of wasted votes
• REG = difference in proportion of wasted votes for each party
• Perfectly fair (0%) = both parties waste same % of votes rather than same number of votes
• Can be used as an adjustment in any example
Relative efficiency gap

\[ \text{REG} = \frac{11}{26} - \frac{9}{19} = 5\% \text{ in favor of } \star (\text{less } \% \text{ of votes wasted}) \]
Relative efficiency gap

\[
\text{REG} = \left( \frac{14}{19} \right) - \left( \frac{6}{26} \right) = 50\% \text{ in favor of } \star \rightarrow +2.5 \text{ seat advantage}
\]

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<tr>
<td>Total</td>
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When EG goes wrong

10 districts of 10 voters each
Each district contains 9 ★ voters and 1 ♦ voter
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10 districts of 10 voters each
Each district contains 9 ★ voters and 1 ♦ voter
Wasted votes in each district: 3★, 1 ♦
EG = (3−1)×10 / 100 = 20% in favor of ♦ → +2 seat advantage for ♦
When EG goes wrong

10 districts of 10 voters each
Each district contains 9 ★ voters and 1 ♦ voter
Wasted votes in each district: 3★, 1 ♦
EG = (3−1)x10 / 100 = 20% in favor of ♦ → +2 seat advantage for ♦
♦ didn’t win any seats so a +2 seat advantage doesn’t make sense…
Mathematical fixes

“Modified” efficiency gap

• Based on numbers, some votes must be wasted!
• Consider “actually wasted” votes: \( (\text{wasted}) - (\text{must waste}) \)
• \( \text{MEG} = \text{difference in “actually wasted” votes between the two parties} \)
• Used in extreme cases where one party has vast majority of voters
Modified efficiency gap

10 districts of 10 voters each (9 ★, 1 ♦ in each)

★ only needs 60 to win all 10 districts → 30 must be wasted!

♦ needs 6 to win 1 district (at most) → 4 must be wasted!
Modified efficiency gap

10 districts of 10 voters each (9 ★, 1 ◆ in each)

★ only needs 60 to win all 10 districts → 30 must be wasted!
◆ needs 6 to win 1 district (at most) → 4 must be wasted!

★ wasted 30 votes but they had to! → ★ actually wasted 0 votes
◆ wasted 10 votes but they had to waste 4 → ◆ actually wasted 6 votes

MEG = (6-0)/100 = 6% in favor of ★ → +0.6 seat advantage for ★ (makes sense!)
Main Question

To what extent can or should we use the outcomes of elections to determine a potential gerrymander?

In other words, is there a measure (quantitative or otherwise) that can be used to test for partisan gerrymandering?
Redistricting Principles

Here are some of the most common principles that states emphasize in new district plans:

- Compactness
- Preserve communities of interest
- *Not in favor of a specific political party*
- Avoid pairing incumbents
- Promote competitive elections