

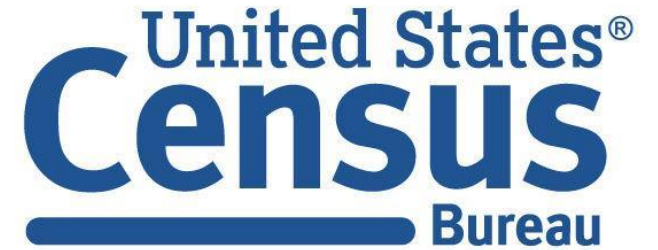
A complex network diagram with various sized nodes (black, blue, and grey) connected by thin grey lines. Some nodes are highlighted with larger circles. The background is white with faint grey circular patterns.

# Mathematics and Redistricting

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Trinity College, Fall 2021 (Kyle Evans)

# The U.S. Census



- **Conducted every 10 years (April 1, xxx0)**
  - In the Constitution, has been done since 1790
  - Represents a snapshot in time
- **Necessary for apportionment, redistricting, distribution of federal funds**
- **2020 was first Census with option to complete online**
- **Mandatory for everyone living in the U.S.**
  - Fines for ignoring or false answers can vary from \$100 to \$5,000 but last enforced in 1970
- **Identifiable Census data is released to the public 72 years after it is completed**
- **American Community Survey (ACS) given to ~3.5 million households/year**
  - Much more detailed information, also mandatory
  - Done by random sampling

# What information is collected?



## Included

- Name
- Age
- Gender
- Race
- Ethnicity
- Relationship
- Own or rent residence


## Not included

- Salary
- Political party affiliation
- Social Security number
- Employment status
- Citizenship status

# The U.S. Census

## The Citizenship Question

Is this person a citizen of the United States?

- Yes, born in the United States
- Yes, born in Puerto Rico, Guam, the U.S. Virgin Islands, or Northern Marianas
- Yes, born abroad of U.S. citizen parent or parents
- Yes, U.S. citizen by naturalization – *Print year of naturalization* 
- No, not a U.S. citizen

# The U.S. Census

## The Citizenship Question

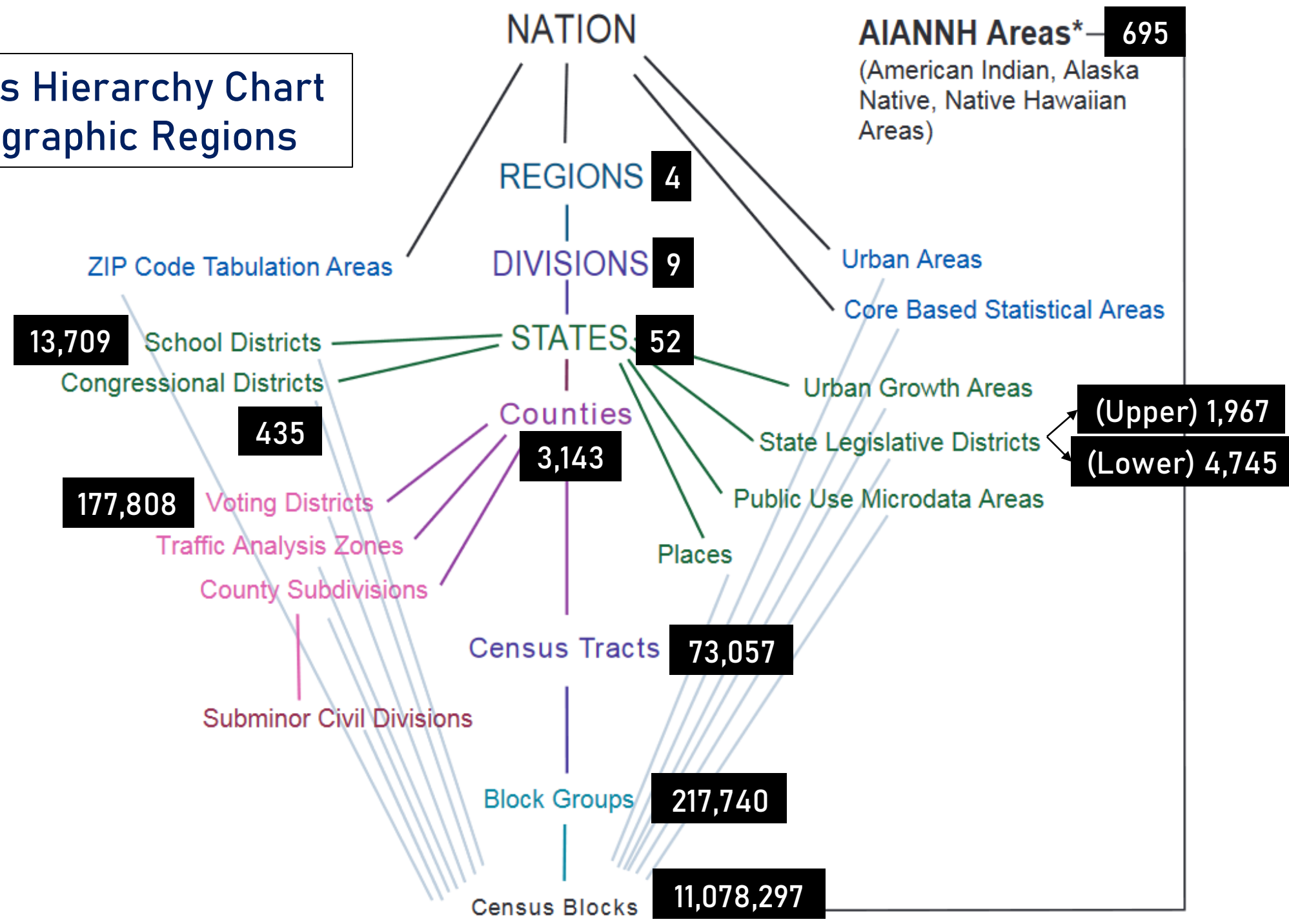
- Census has always counted every person living in the U.S. (citizens, legal noncitizens, unauthorized residents)
- Question was last asked on Census in 1950 (still asked on ACS)
- Trump administration wanted to instill fear, mainly among Latinos, to undercount potential Democrat voters and benefit Republicans in the apportionment counts and state funding
- *New York v. U.S. Dept. of Commerce (2019)* – Supreme Court ruled that the question would not be included as it would undermine accuracy and be discriminatory

Is this person a citizen of the United States?

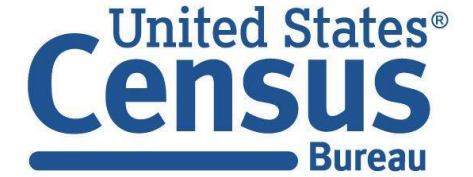
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# Census Hierarchy Chart of Geographic Regions



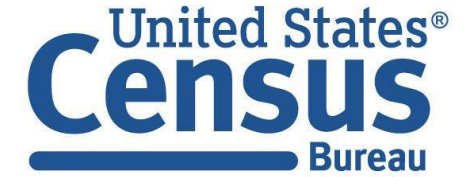
# How are different groups counted?



- College students in dorms?      Address of dorm

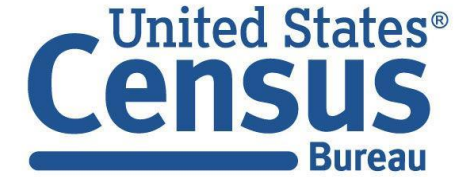


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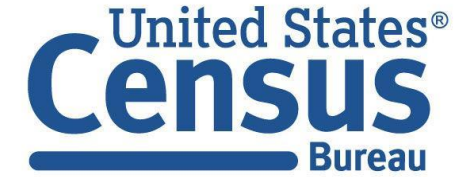
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- College students in dorms? Address of dorm
- Overseas military? Home address in U.S.

# How are different groups counted?



- College students in dorms? Address of dorm
- Overseas military? Home address in U.S.
- Nursing homes?

# How are different groups counted?



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- Foreigners on vacation?

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- Foreigners on vacation? Not counted

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- Foreigners on vacation? Not counted
- Homelessness?

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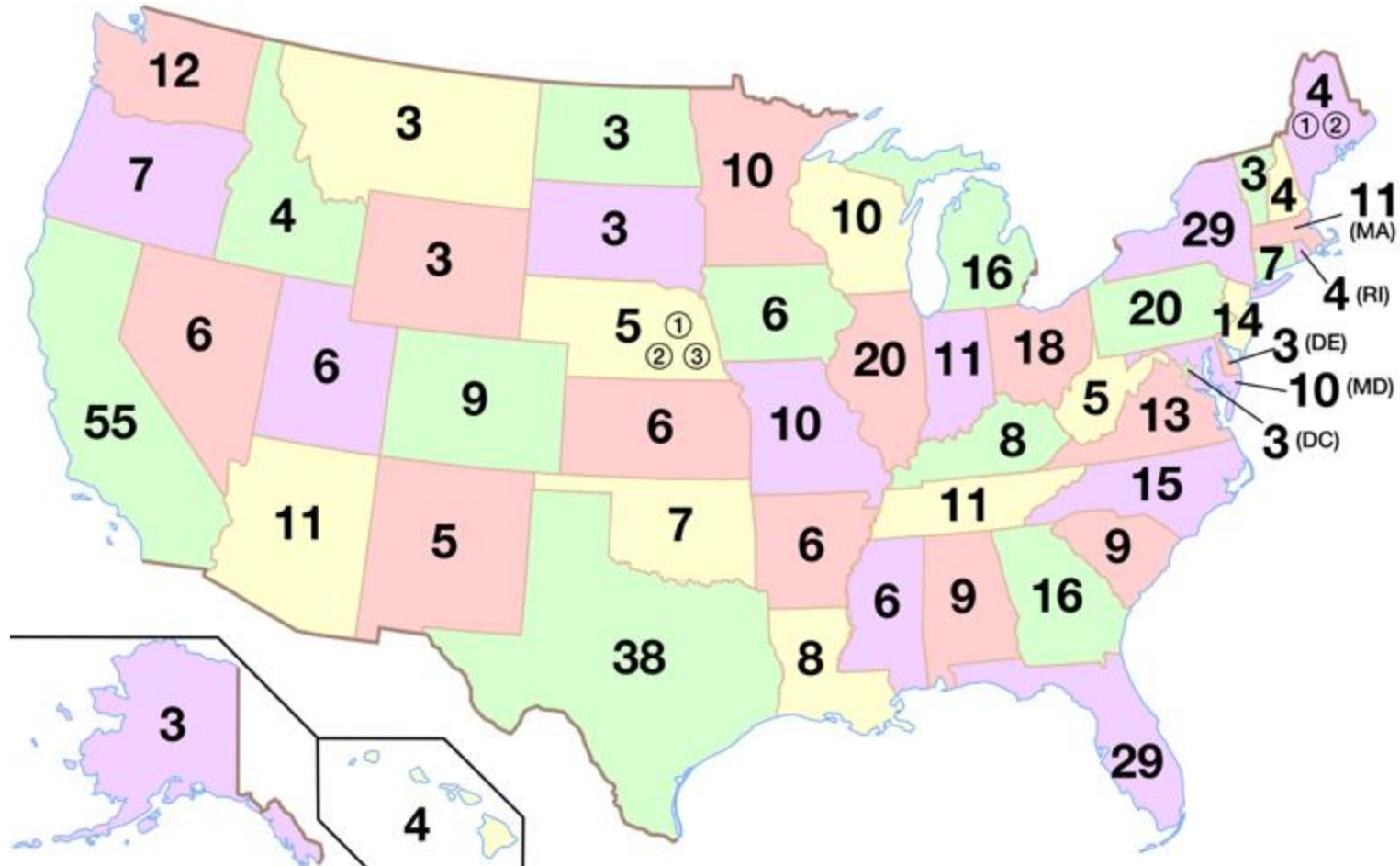
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- Foreigners on vacation? Not counted
- Homelessness? Shelters / soup kitchens / outside

# Differential Privacy

- The more accurate data is, the easier it is to identify individuals
  - Privacy/confidentiality an issue now more than ever with “big data” / technology
  - Trade-off between accuracy and privacy
- Old method: “Data Swapping”
  - Example: A census block of 20 people with 1 Filipino-American, their data could be swapped with someone from another block with more Filipino-Americans
- New method for 2020: “Differential Privacy”
  - Keep state total populations as reported, but “inject noise” into census blocks
  - Concerns about accuracy of race and ethnicity data
  - Concerns about population equality of state legislative districts

# Apportionment

This map shows number of Electoral Votes, subtract by 2 to get congressional districts.





# Apportionment Methods

- There have been 4 different mathematical methods used for apportionment in our country's history
  - Hamilton's Method (1850 and 1880)
  - Jefferson's Method (1792-1830)
  - Webster's Method (1840-1930)
  - Huntington-Hill Method (1941-Present)
- First ever Presidential veto was about which method to use (1792)
- 1941 – Congress passed law that fixed House size at 435 representatives and which mathematical method would be used (Huntington-Hill)

# Hamilton's Method

- Step 1: Standard Divisor = (Total population)  $\div$  (Number of representatives)

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Add up the lower quotas, how many seats remain?

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Add up the lower quotas, how many seats remain?

- Step 4: Fractional Part = number following the decimal of the standard quota

Assign states extra seats starting with largest fractional part

Final representatives = (Lower Quota) or (Lower Quota + 1)

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- Step 2: Choose a Modified Divisor (slightly less than the standard divisor) and for each state:  $\text{Modified Quota} = (\text{State Population}) \div (\text{Modified Divisor})$
- Step 3: Check if the new Lower Quotas add up to the desired number of representatives. If not, try a different Modified Divisor.
  - If you still have representatives left over, choose a slightly lower modified divisor.
  - If you assigned too many representatives, choose a slightly higher modified divisor.



# Webster's Method

- Step 1: Calculate the Standard Divisor, Standard Quotas, and Natural Quotas (round naturally based on .5 instead of always rounding down)

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- Step 1: Calculate the Standard Divisor, Standard Quotas, and Natural Quotas (round naturally based on .5 instead of always rounding down)

Check how many representatives have been apportioned.

If rounding naturally adds up to the desired number, process is complete!

- Step 2: Choose a Modified Divisor (adjust for needing to gain or lose reps) and for each state: Modified Quota = (State Population) ÷ (Modified Divisor)
- Step 3: Check if the new Natural Quotas add up to the desired number of representatives. If not, try a different Modified Divisor.
  - If you need to gain representatives, choose a slightly lower modified divisor.
  - If you assigned too many representatives, choose a slightly higher modified divisor.

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- Step 2: Round Standard Quotas according to the Geometric Mean of the lower and upper quotas.

$$\text{Geometric Mean of } a \text{ and } b = \sqrt{ab}$$

Check how many representatives have been apportioned.

Example: Suppose a state has a standard quota of 4.481

The geometric mean of 4 and 5 (lower and upper quotas) is  $\sqrt{20} \approx 4.472$

$4.481 > 4.472 \rightarrow$  round standard quota up to 5

If standard quota  $<$  geometric mean  $\rightarrow$  round down

# Huntington-Hill Method

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Check how many representatives have been apportioned.

- Step 3: Choose a Modified Divisor and round Modified Quotas according to the geometric means until the correct number of representatives are assigned.

# Population Changes 2010-2020 (Projected)

## Largest Population Increases

- Utah (+18%)
- Idaho (+17%)
- Texas (+16%)
- North Dakota (+16%)
- Washington D.C. (+15%)
- Nevada (+15%)
- Colorado (+15%)

## Population Decreases

- West Virginia (-3%)
- Mississippi (-0.2%)
- Illinois (-0.1%)

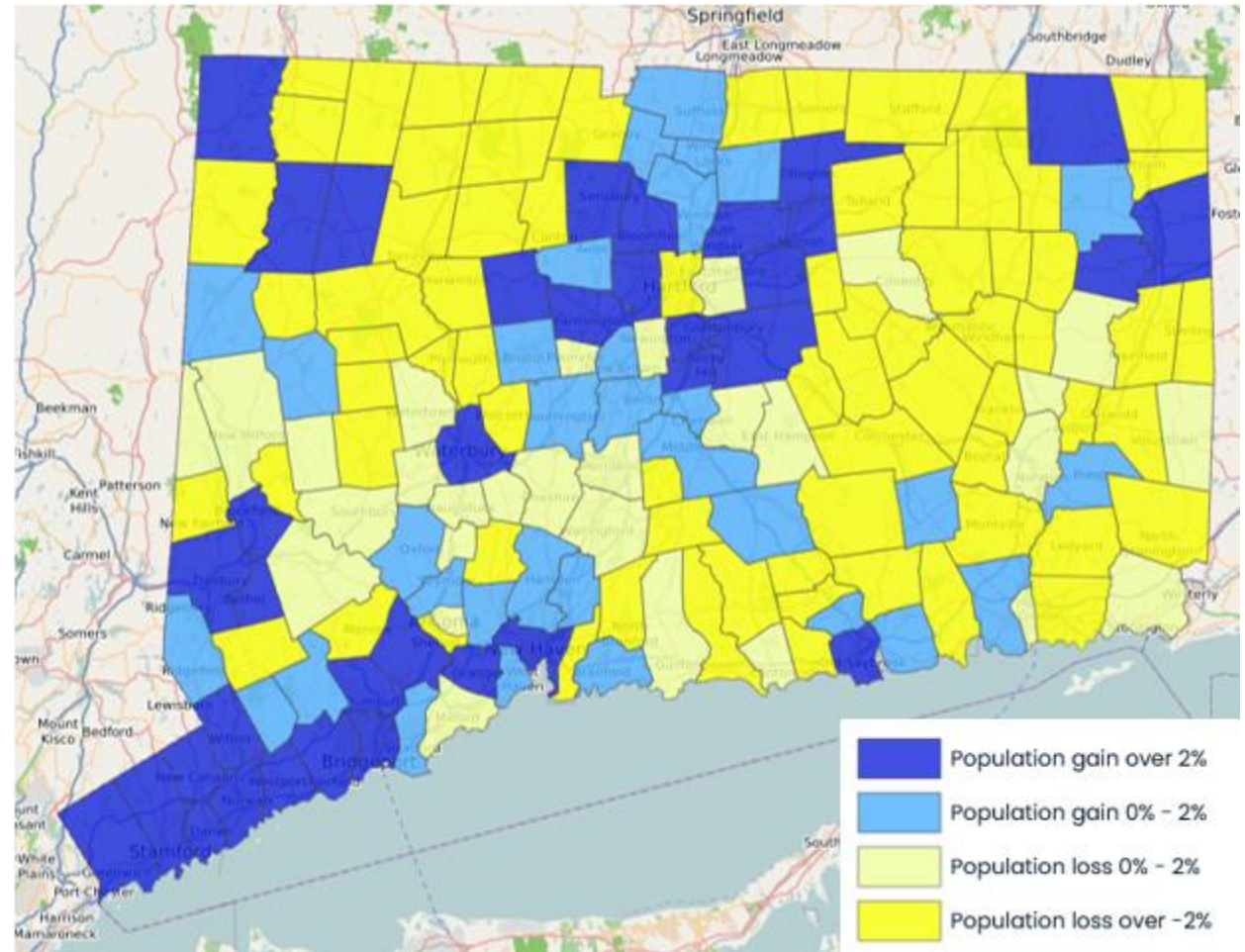
## Smallest Increases

- Connecticut (+0.9%)
- Michigan (+2%)
- Wyoming (+2%)

# Connecticut Census Changes

- Total Pop. = 3,605,944
- Stamford gained the most people

Racial Group	2010	2020
White	77.6%	66.4% (-377,282)
Hispanic or Latino	13.4%	17.3% (+144,206)
Black	10.1%	10.8% (+25,818)
Asian	3.8%	4.8% (+36,368)

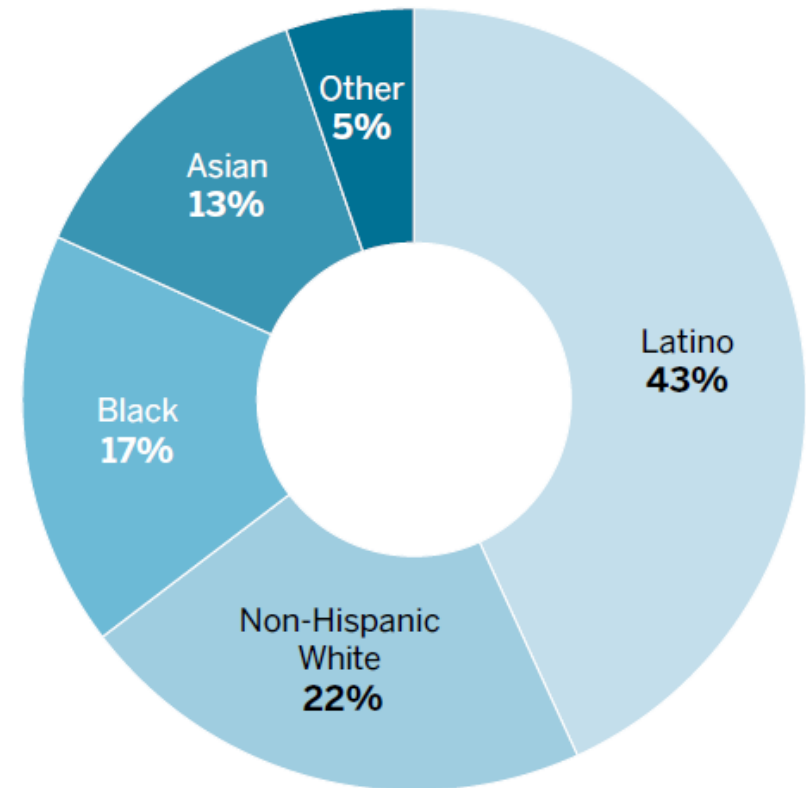




# Factors Impacting Population Changes

- Births/deaths
- Immigration
- In- and out-migration (between states)
  - In-migration (SC, MT, NV, OR, TN)
  - Out-migration (NYC, Northeast)
- Demographic changes
  - 78% of population increase is non-white

**Estimated Makeup of Increase  
in Eligible U.S. Voters, 2010–20**

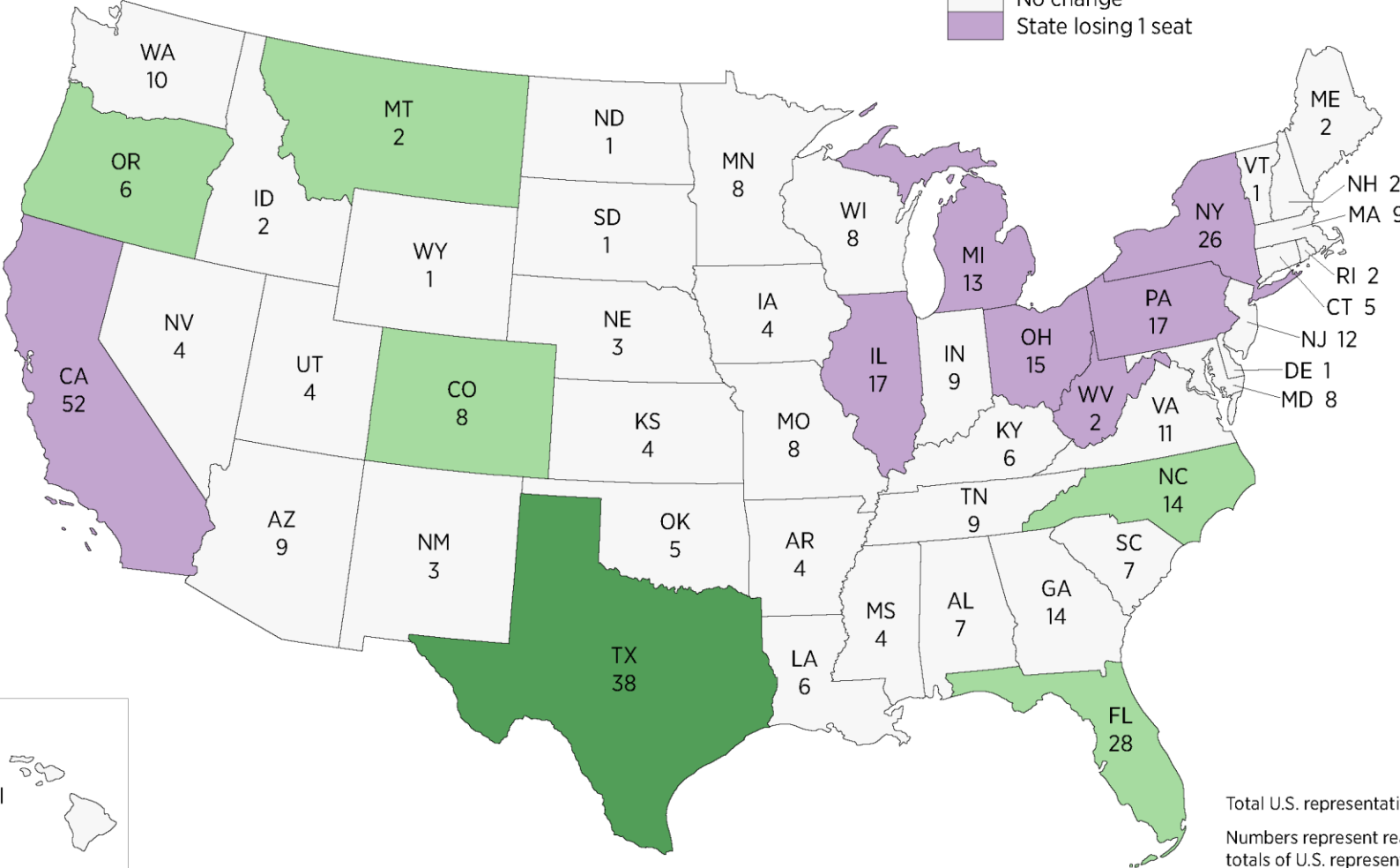


# Apportionment of the U.S. House of Representatives Based on the 2020 Census



**Change from 2010 to 2020**

- State gaining 2 seats
- State gaining 1 seat
- No change
- State losing 1 seat

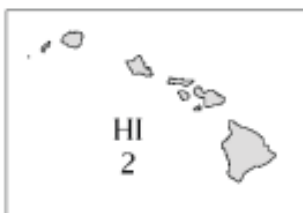
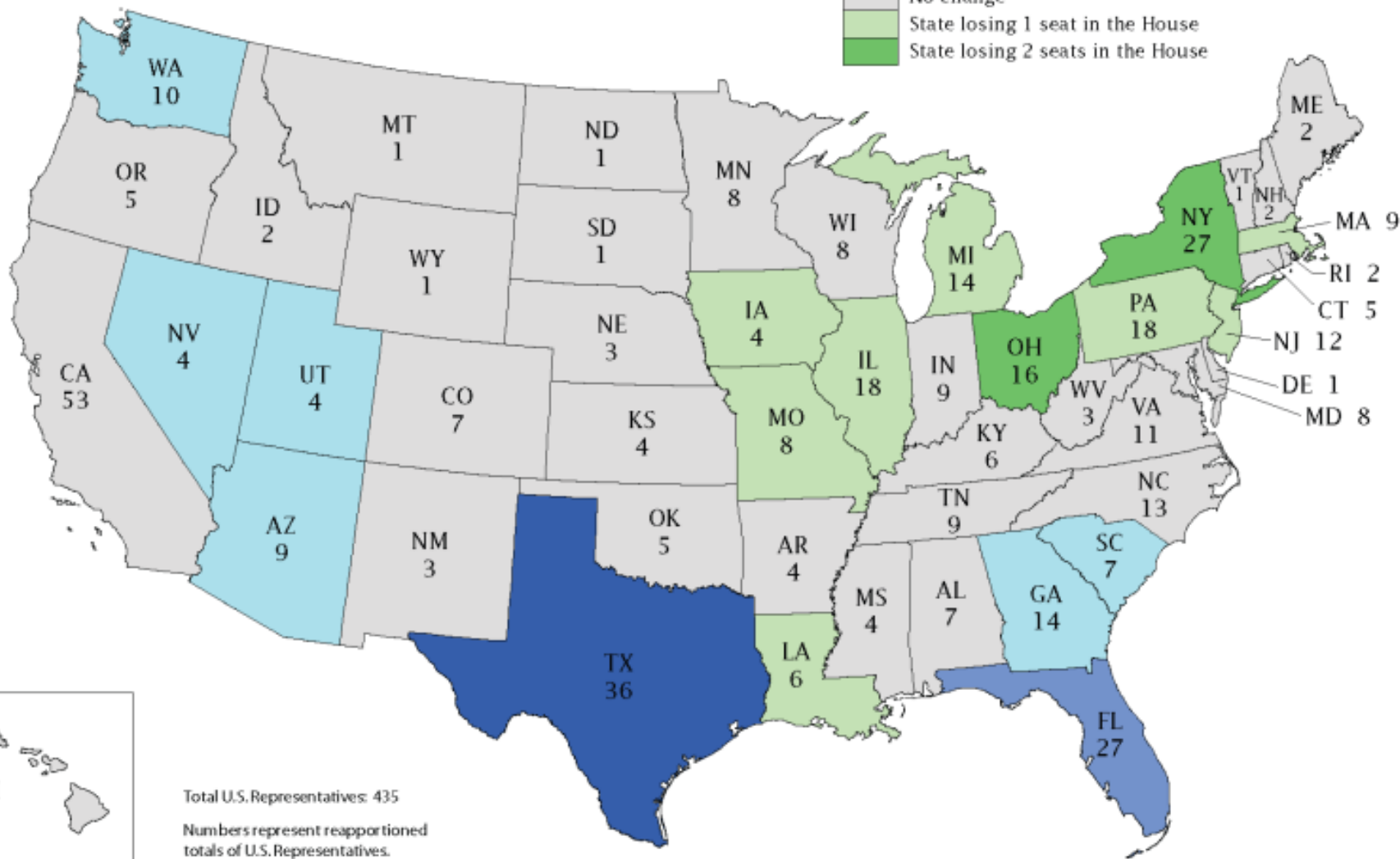


Total U.S. representatives: 435  
 Numbers represent reapportioned totals of U.S. representatives.

# Apportionment of the U.S. House of Representatives Based on the 2010 Census

Change from 2000 to 2010

- State gaining 4 seats in the House
- State gaining 2 seats in the House
- State gaining 1 seat in the House
- No change
- State losing 1 seat in the House
- State losing 2 seats in the House



Total U.S. Representatives: 435  
Numbers represent reapportioned totals of U.S. Representatives.