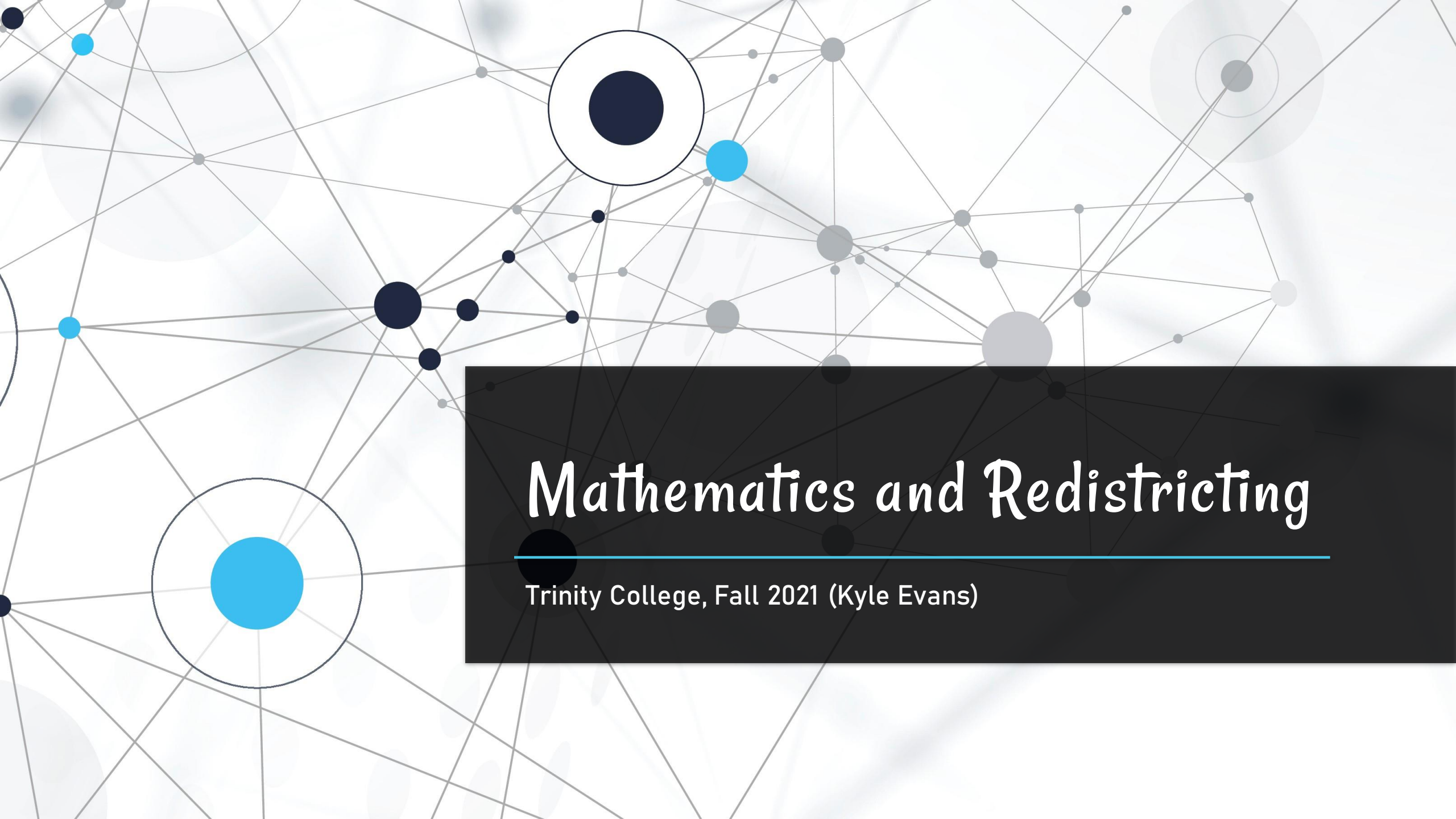


Mathematics and Redistricting

Trinity College, Fall 2021 (Kyle Evans)

The Commission considered statewide state and federal partisan general election results during the last ten years. There were sixteen such contests. When considering the results of each of those elections, the Commission determined that Republican candidates won thirteen out of sixteen of those elections resulting in a statewide proportion of voters favoring statewide Republican candidates of 81% and a statewide proportion of voters favoring statewide Democratic candidates of 19%. When considering the number of votes cast in each of those elections for Republican and Democratic candidates, the statewide proportion of voters favoring statewide Republican candidates is 54% and the statewide proportion of voters favoring statewide Democratic candidates is 46%. Thus, the statewide proportion of voters favoring statewide Republican candidates is between 54% and 81% and the statewide proportion of voters favoring statewide Democratic candidates is between 19% and 46%. The Commission obtained publicly available geographic data for statewide partisan elections in 2016, 2018, and 2020. Publicly available geographic data for those elections was not available for elections in 2012 and 2014. Using this data, the Commission adopted the final general assembly district plan, which contains 85 districts (64.4%) favoring Republican candidates and 47 districts (35.6%) favoring Democratic candidates out of a total of 132 districts. Accordingly, the statewide proportion of districts whose voters favor each political party corresponds closely to the statewide preferences of the voters of Ohio.

Population Deviation

- We know that districts are required to be equal to preserve the principle of "one person, one vote", but is some deviation allowed?
- Congressional districts must be "as nearly equal as practicable"
  - Established in *Wesberry v. Sanders* (1964) – a GA district had nearly 3x population as others
  - A New Jersey plan with average deviation of 0.14% was rejected in *Karcher v. Daggett* (1983)
- Legislative districts require "substantial equality" but have more flexibility
  - Courts have allowed a 10% deviation between the largest and smallest districts
  - Some deviation can preserve communities of interest, avoid splitting counties, etc.

Population Deviation

Suppose we have 100,000 people to be divided into 10 districts

- Ideal population – total population ÷ number of districts

(100,000 / 10) = 10,000

Population Deviation

Suppose we have 100,000 people to be divided into 10 districts

- Ideal population – total population ÷ number of districts

(100,000 / 10) = 10,000

- District deviation – difference from ideal as a percentage

A district with 10,200 has a deviation of (200 / 10,000) = 2%

Population Deviation

Suppose we have 100,000 people to be divided into 10 districts

- Ideal population – total population ÷ number of districts

(100,000 / 10) = 10,000

- District deviation – difference from ideal as a percentage

A district with 10,200 has a deviation of (200 / 10,000) = 2%

- Average deviation – sum of all deviations ÷ number of districts

All deviations should be treated as positive numbers

(2% + 1% + 1%) / 3 = 1.33% average

Population Deviation

Suppose the largest district has a population of 10,200 and the smallest district has a population of 9,850

- Deviation range – the difference in percent deviation between the largest and smallest districts

Population Deviation

Suppose the largest district has a population of 10,200 and the smallest district has a population of 9,850

- Deviation range – the difference in percent deviation between the largest and smallest districts

Largest: (200 / 10,000) = 2% Smallest: (-150 / 10,000) = -1.5%

Deviation range = 2% + 1.5% = 3.5%

Maximum range that can be possibly allowed:

Congressional plans: 0.75%

Legislative plans: 10%

Partisan Gerrymandering Measures

How can we attempt to quantify how “unfair” a political party’s advantage is a district map?

For example, 26★ and 19♦ into 5 districts of 9 each

We know ★ can possibly win all 5 districts with only 58% of voters

This can only be done by cracking, but how can we measure the advantage held by the ★ party?

What is the efficiency gap?

- A quantitative measure to define the extent of partisan gerrymandering
- Impacted the initial court ruling in *Gill v. Whitford* (2016)
  - Democrats had "trifecta" in before 2010 elections, Republicans had trifecta after 2010 elections and got to control redistricting process
- District Court ruled that Wisconsin had to redraw State House map by 2017
- Appeal to Supreme Court → sent back to lower courts to decide after SC ruled that partisan gerrymandering was to be determined by the states

What is the efficiency gap?

Developed by Stephanopoulos / McGhee in 2014

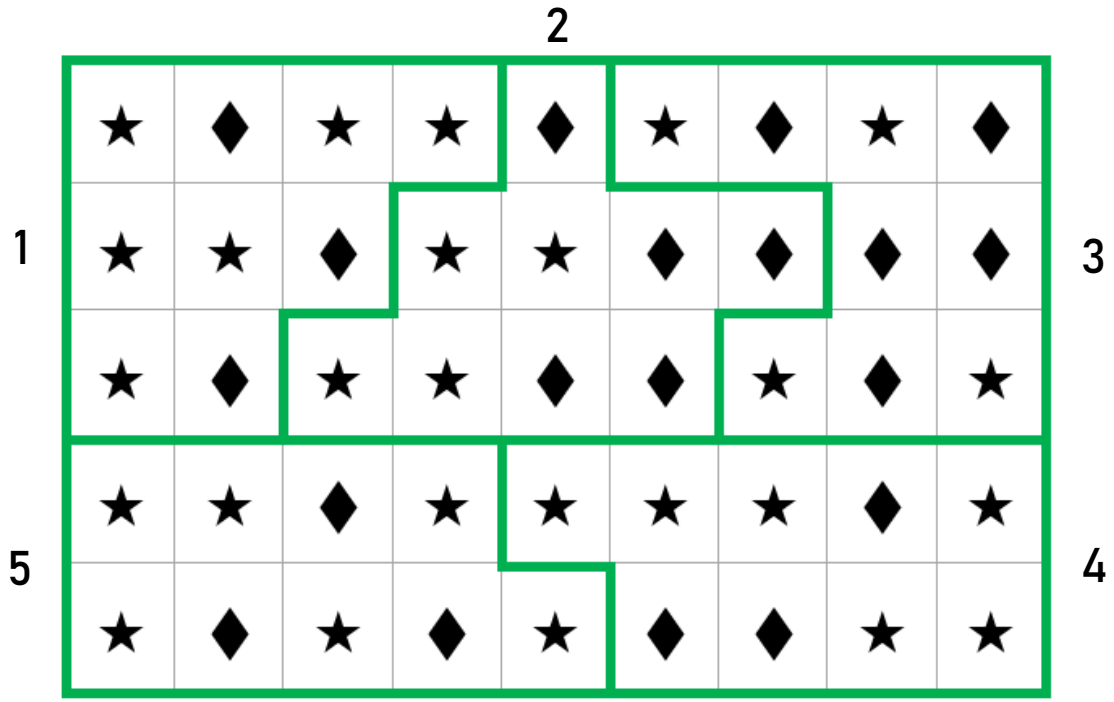
Based on the concept of “wasted votes” and designed to measure the extent of packing and cracking of voters

- For a winning party, any votes above those needed to win are “wasted”
- For a losing party, any votes are “wasted”
- Calculation is based on which party “wastes” more votes than the other
- Efficiency gap = (difference in wasted votes) / total votes (x 100%)

What is the efficiency gap?

Example:

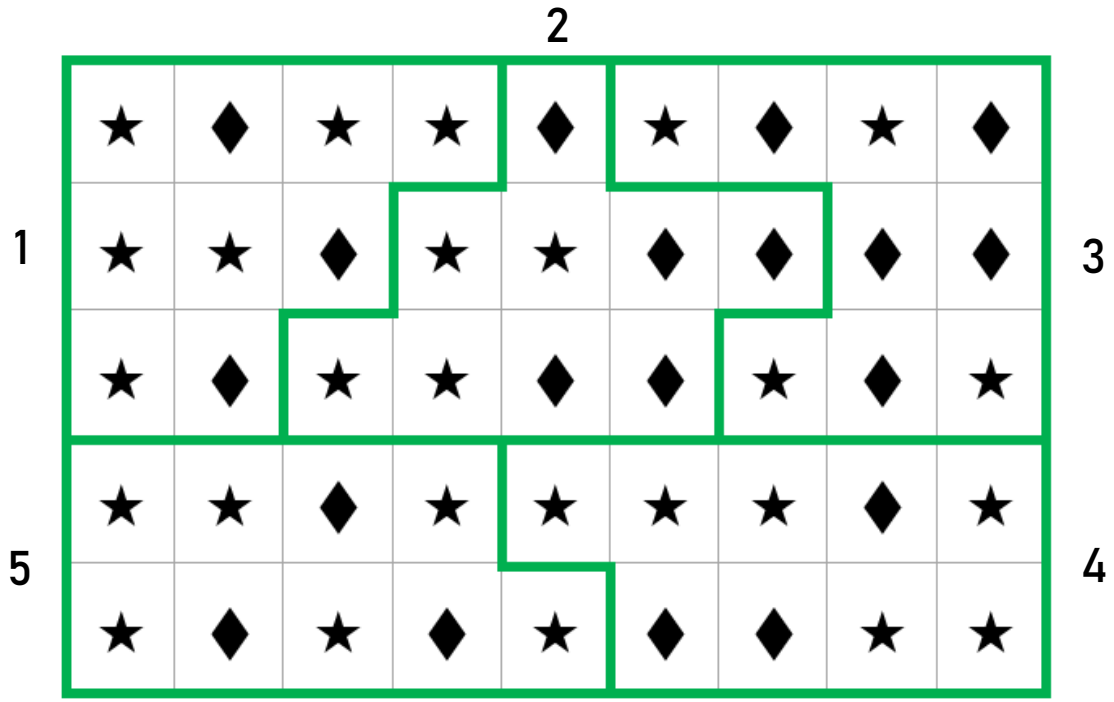
District	★ wasted	♦ wasted
1		
2		
3		
4		
5		



What is the efficiency gap?

Example:

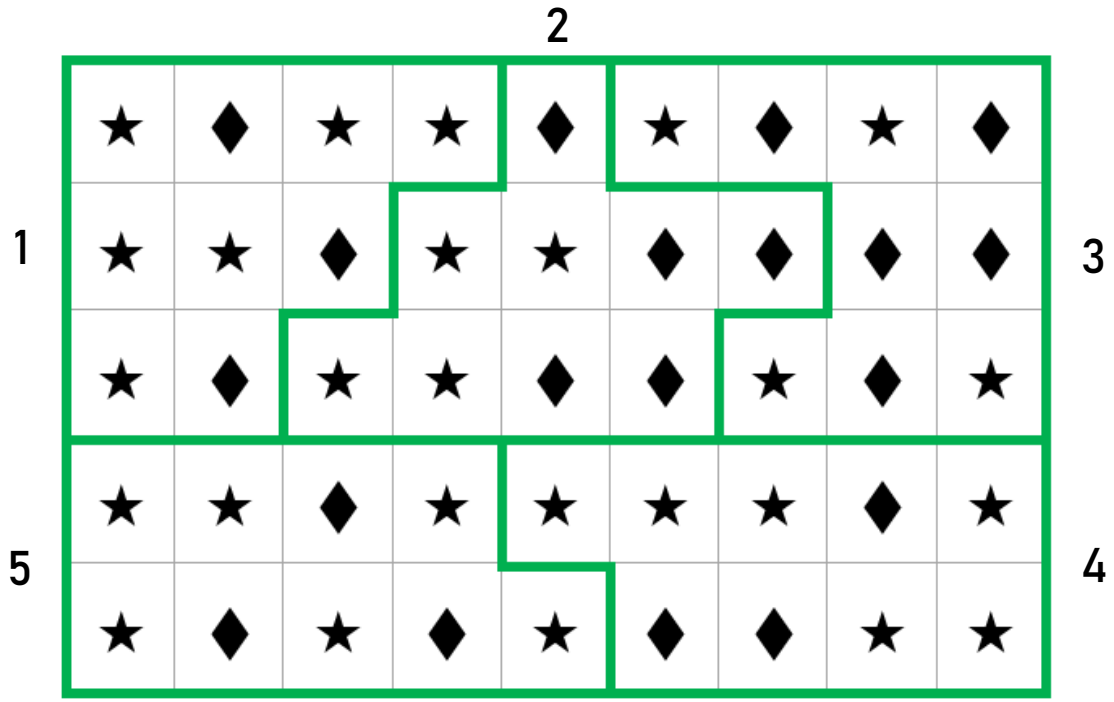
District	★ wasted	♦ wasted
1	1	3
2		
3		
4		
5		



What is the efficiency gap?

Example:

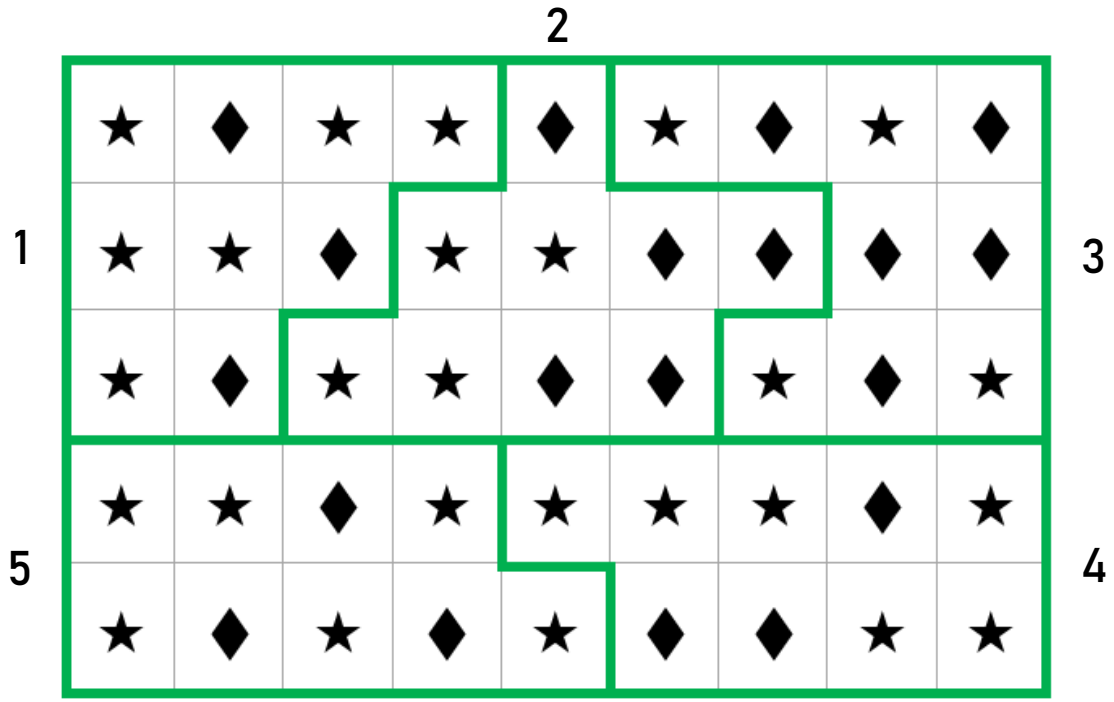
District	★ wasted	♦ wasted
1	1	3
2	4	0
3		
4		
5		



What is the efficiency gap?

Example:

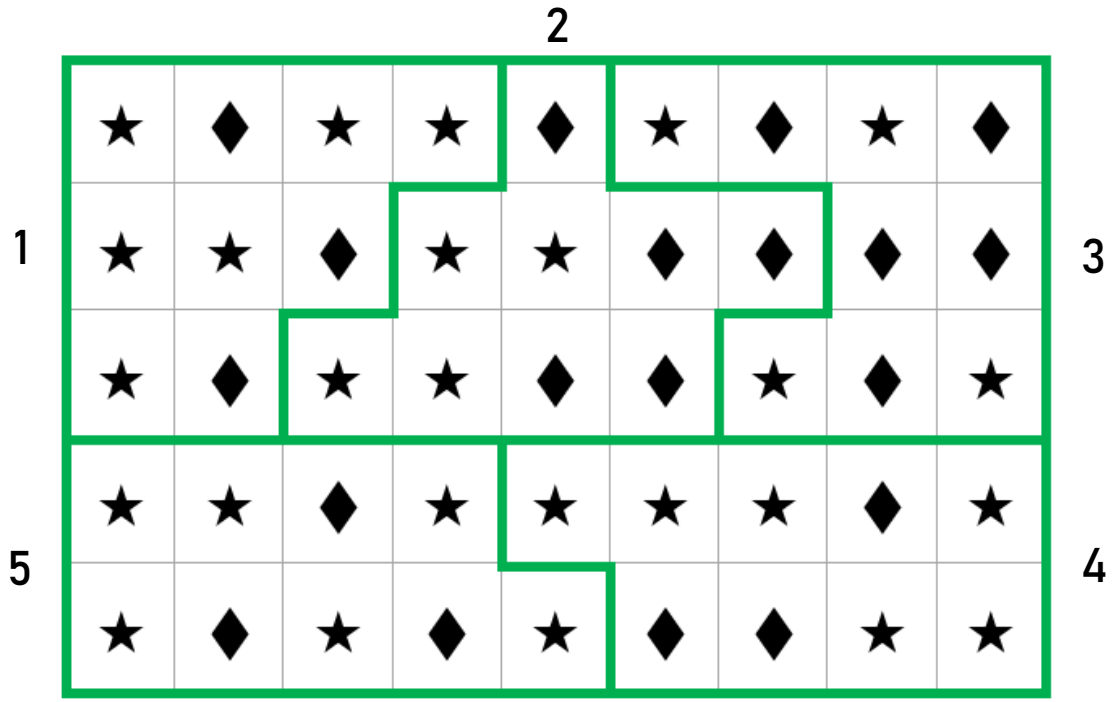
District	★ wasted	♦ wasted
1	1	3
2	4	0
3	4	0
4	1	3
5	1	3



What is the efficiency gap?

Example:

District	★ wasted	♦ wasted
1	1	3
2	4	0
3	4	0
4	1	3
5	1	3
Total	11	9



EG = (11-9)/45 = 4% in favor of ♦ (less votes wasted)

Interpreting the efficiency gap

What does the final calculation mean?

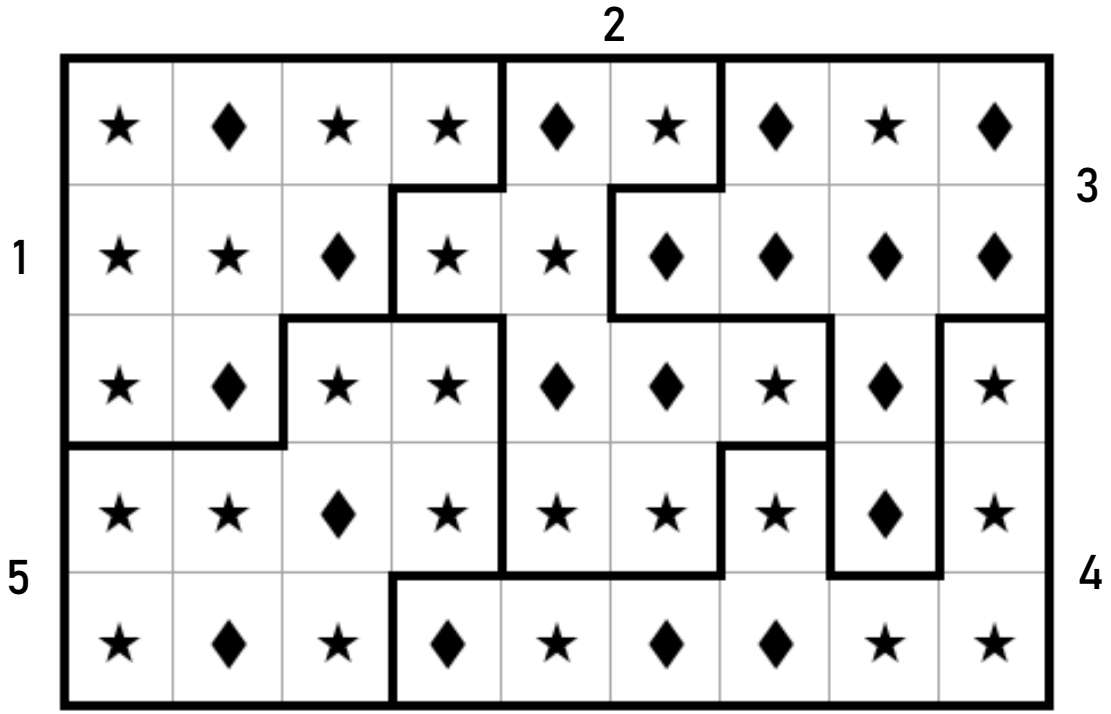
- 0% means each wasted the same number of votes → perfectly fair
- The higher the %, the more unfair
- When multiplied by # of districts, % gives us how much of a seat advantage
- From example: 4% × 5 districts = 0.2 seat advantage for ♦

What is considered "too unfair"?

- Congressional district plans: +2 (or more) seats
- State district plans: +8% (or more)

Efficiency gap example

District	★ wasted	♦ wasted
1	1	3
2	1	3
3	1	3
4	1	3
5	2	2
Total	6	14



EG= (14-6)/45 = 18% in favor of ★ → +1 seat advantage

Alternative Formula

When all districts have the same number of voters, the efficiency gap has a shortcut formula:

$$\text{Efficiency Gap} = (\%\ \text{of Districts won} - 50\%) - 2 \times (\%\ \text{of Votes} - 50\%)$$

Alternative Formula

When all districts have the same number of voters, the efficiency gap has a shortcut formula:

Efficiency Gap = (% of Districts won – 50%) – 2 x (% of Votes – 50%)

In example #1, ★ wins 3/5 = 60% of districts and has 26/45 = 58% of votes

EG = (60% - 50%) – 2 x (58% - 50%) = -6% → 6% advantage for ♦

Alternative Formula

When all districts have the same number of voters, the efficiency gap has a shortcut formula:

Efficiency Gap = (% of Districts won – 50%) – 2 x (% of Votes – 50%)

In example #1, ★ wins 3/5 = 60% of districts and has 26/45 = 58% of votes

EG = (60% - 50%) – 2 x (58% - 50%) = -6% → 6% advantage for ♦

In example #2, ★ wins 4/5 = 80% of districts and has 26/45 = 58% of votes

EG = (80% - 50%) – 2 x (58% - 50%) = 14% advantage for ★

Alternative Formula

Efficiency Gap = (% of Districts won – 50%) – 2 x (% of Votes – 50%)

If one party receives 60% of the votes, then according to the efficiency gap formula, the fairest map is one in which that party wins what % of districts?

Alternative Formula

Efficiency Gap = (% of Districts won – 50%) – 2 x (% of Votes – 50%)

If one party receives 60% of the votes, then according to the efficiency gap formula, the fairest map is one in which that party wins what % of districts?

$$(x - 50\%) - 2 \times (60\% - 50\%) = 0$$

$$x = 70\% \text{ of districts}$$

Alternative Formula

Efficiency Gap = (% of Districts won – 50%) – 2 x (% of Votes – 50%)

If one party receives 60% of the votes, then according to the efficiency gap formula, the fairest map is one in which that party wins what % of districts?

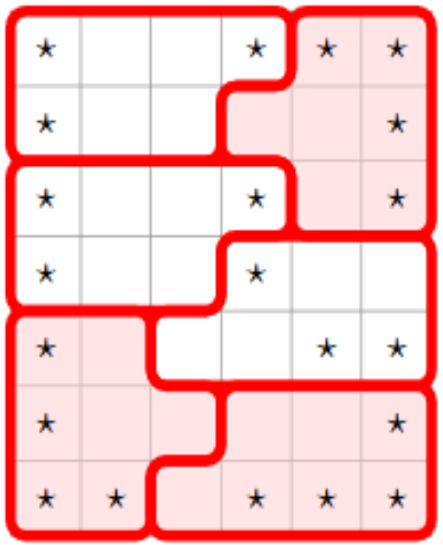
$$(x - 50\%) - 2 \times (60\% - 50\%) = 0$$

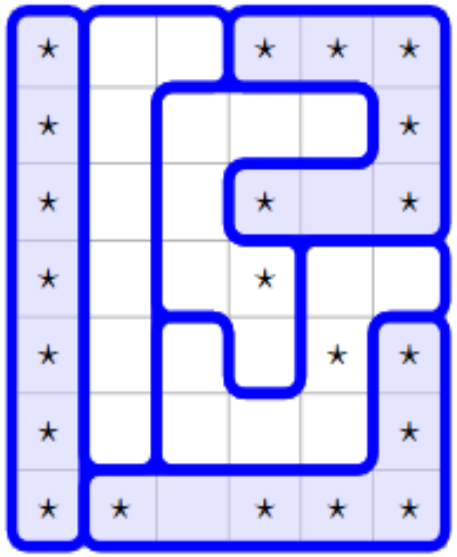
$$x = 70\% \text{ of districts}$$

In other words, proportional representation isn't always considered fair!

Efficiency gap examples

What is similar and different about the two district plans below?





Efficiency gap examples

We conclude that the efficiency gap of an entire districting plan only depends on the number of districts won, not how they are won.

The two examples have 3 district wins in very different ways, but the overall number of wasted votes are identical.

This begs the question about the usefulness of the formula…

Criticisms of the efficiency gap

- Uses election results, but redistricting comes before elections
- “Judging fairness from 1 election is like judging fairness of a coin after 1 flip”
- Wasted votes would be like saying “wasted runs” in baseball
- +2 seat advantage much more likely in larger states
- Ignores competitive elections
- Why is 75/25 the “most fair” district outcome?
- Chief Justice Roberts: “sociological gobbledygook“
- Doesn’t always make sense mathematically!

Efficiency gap in practice

- 2018 – Missouri became the first state to amend its Constitution to explicitly address the efficiency gap, stating that legislative plans must aim to get this value as close to zero as possible
  - Ballot initiative that passed also included a new position of “state demographer”

Efficiency gap in practice

- 2018 – Missouri became the first state to amend its Constitution to explicitly address the efficiency gap, stating that legislative plans must aim to get this value as close to zero as possible
  - Ballot initiative that passed also included a new position of “state demographer”
- 2020 – Missouri voters passed a new ballot initiative that essentially eliminated redistricting reform from 2018
  - State demographer replaced with political commissions
  - Attempt to only count voting-age adults
  - Public transparency and input still exist

When EG goes wrong

10 districts of 10 voters each

Each district contains 9 ★ voters and 1 ♦ voter

When EG goes wrong

10 districts of 10 voters each

Each district contains 9 ★ voters and 1 ♦ voter

Wasted votes in each district: 3★, 1 ♦

EG = (3-1)x10 / 100 = 20% in favor of ♦ → +2 seat advantage for ♦

When EG goes wrong

10 districts of 10 voters each

Each district contains 9 ★ voters and 1 ♦ voter

Wasted votes in each district: 3★, 1 ♦

EG = (3-1)x10 / 100 = 20% in favor of ♦ → +2 seat advantage for ♦

♦ didn't win any seats so a +2 seat advantage doesn't make sense…

Mathematical fixes

“Modified” efficiency gap

- Based on numbers, some votes must be wasted!
- Consider “actually wasted” votes: (wasted) – (must waste)
- MEG = difference in “actually wasted” votes between the two parties
- Used in extreme cases where one party has vast majority of voters

Modified efficiency gap

10 districts of 10 voters each (9 ★, 1 ♦ in each)

- ★ only needs 60 to win all 10 districts → 30 must be wasted!
- ♦ needs 6 to win 1 district (at most) → 4 must be wasted!

Modified efficiency gap

10 districts of 10 voters each (9 ★, 1 ♦ in each)

- ★ only needs 60 to win all 10 districts → 30 must be wasted!
- ♦ needs 6 to win 1 district (at most) → 4 must be wasted!

- ★ wasted 30 votes but they had to! → ★ actually wasted 0 votes
- ♦ wasted 10 votes but they had to waste 4 → ♦ actually wasted 6 votes

MEG = (6-0)/100 = 6% in favor of ★→ +0.6 seat advantage for ★(makes sense!)

Main Question

To what extent can or should we use the outcomes of elections to determine a potential gerrymander?

In other words, is there a measure (quantitative or otherwise) that can be used to test for partisan gerrymandering?

Redistricting Principles

Here are some of the most common principles that states emphasize in new district plans:

- Compactness
- Preserve communities of interest
- *Not in favor of a specific political party*
- Avoid pairing incumbents
- Promote competitive elections